Impact of the open-loop gain in a type 2 compensator
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Type 2 specifications:

$G_{MB} := 10 \ \ dB$ Mid-band gain $G_{fc} := 10^{\frac{G_{MB}}{20}} = 3.162$

$f_c := 15kHz$ Crossover frequency for mid-band gain value

$Boost := 65°$ Wanted phase boost at crossover

$$f_p := \left(\tan(Boost) + \sqrt{\tan(Boost)^2 + 1}\right)\cdot f_c = 67.661\cdot kHz$$

$$f_z := \frac{f_c^{\,2}}{f_p} = 3.325\cdot kHz$$

Type 2 components:

$R_1 := 38k\Omega$ This is the upper resistance

$$R_2 := \frac{R_1\cdot f_p\cdot G_{fc}}{f_p - f_z}\cdot\sqrt{\frac{\left(\frac{f_c}{f_p}\right)^2 + 1}{\left(\frac{f_z}{f_c}\right)^2 + 1}} = 126.378\cdot k\Omega$$

$$C_1 := \frac{1}{2\pi\cdot R_2\cdot f_z} = 0.379\cdot nF \qquad C_2 := \frac{C_1}{2\cdot\pi\cdot f_p\cdot C_1\cdot R_2 - 1} = 0.02\cdot nF$$

Operational amplifier specs:

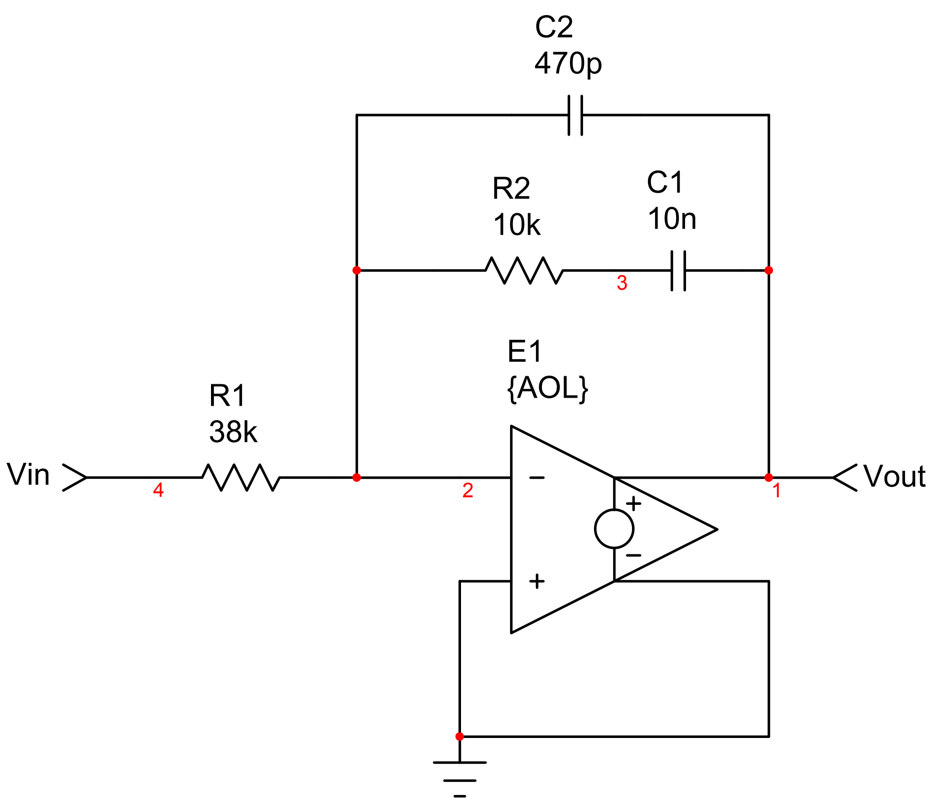
$A_{OLdB} := 70 \ \ dB$ from the op amp data-sheet, open loop gain in dB

$$A_{OL} := 10^{\frac{A_{OLdB}}{20}} = 3.162\times 10^3 \qquad LFP := 30Hz \qquad HLP := 1MHz$$

$$\omega_{pLF} := LFP\cdot 2\cdot\pi \qquad \omega_{pHF} := HLP\cdot 2\cdot\pi$$

$$GBW := A_{OL}\cdot LFP = 0.095\cdot MHz$$

$$G_{op}(s) := A_{OL}\cdot\frac{1}{\left(1 + \frac{s}{\omega_{pLF}}\right)\cdot\left(1 + \frac{s}{\omega_{pHF}}\right)}$$ This is the open-loop op amp transfer function



Type 2 corresponding pole, zero and mid-band gain, calculation check:

$$G_0 := \frac{R_2}{R_1}\cdot\frac{C_1}{C_1 + C_2} \qquad \omega_z := \frac{1}{R_2\cdot C_1} \qquad \omega_p := \frac{1}{R_2\cdot\frac{C_1\cdot C_2}{C_1 + C_2}}$$

midband gain zero pole

$20\cdot\log(G_0) = 10\ \text{dB}$ $f_{z1} := \frac{\omega_z}{2\pi} = 3.325\cdot\text{kHz}$ $f_{p1} := \frac{\omega_p}{2\pi} = 67.661\cdot\text{kHz}$ $f_{peak} := \sqrt{f_z\cdot f_p} = 15\cdot\text{kHz}$

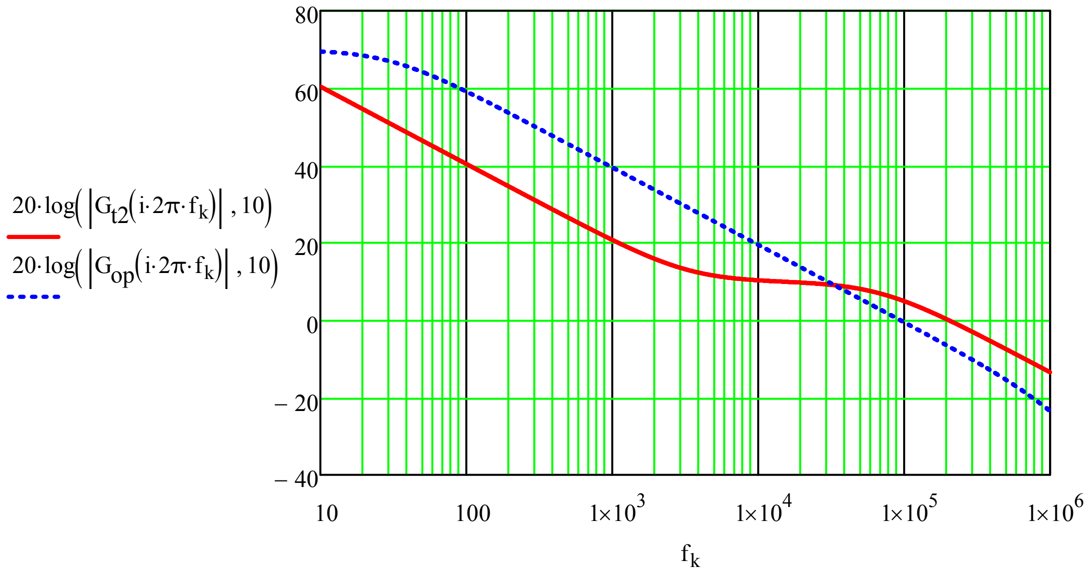
type 2 original transfer function - perfect op amp

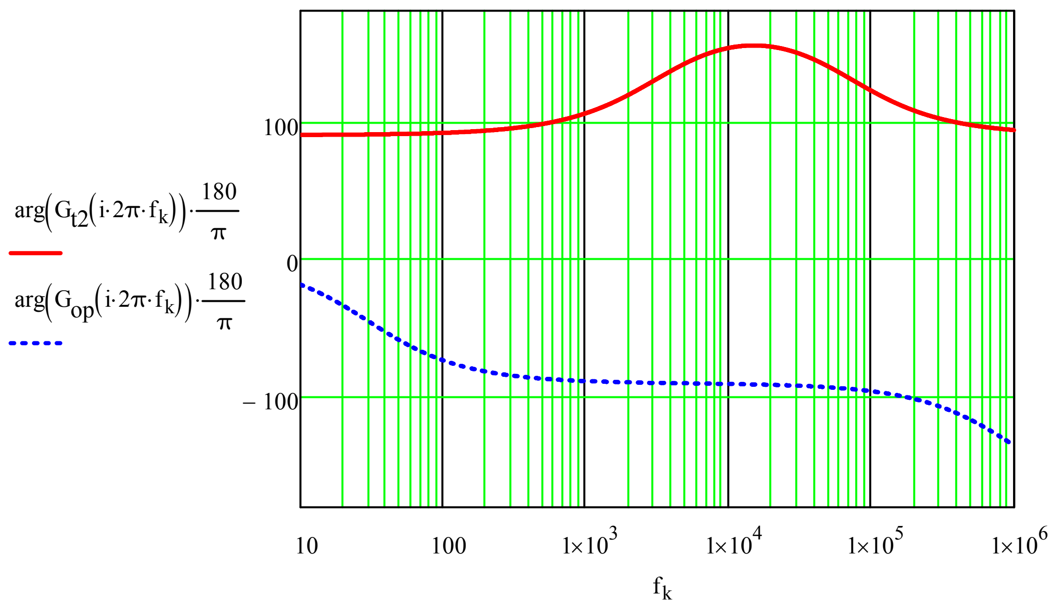
$$G_{t2}(s) := -G_0\cdot\frac{1 + \frac{\omega_z}{s}}{1 + \frac{s}{\omega_p}}$$

Effective boost at crossover: $\arg(G_{t2}(i\cdot 2\pi\cdot f_c)) - 90° = 65\cdot°$

Log frequency sweep

Dynamic response of the Type 2 compensator with perfect op amp
Open-loop plot of the op amp accounting for AOL and poles





Calculations now including the non-perfect op amp characteristics:

open-loop gain AOL
low-frequency pole omegaLF
high-frequency pole omegaHF

$$V_{out} = \varepsilon \cdot A_{OL} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{pLF}}\right)\cdot\left(1 + \frac{s}{\omega_{pHF}}\right)}$$

$$\varepsilon = V_{plus} - V_{minus} = -V_{minus}$$

$$\varepsilon = -\left(V_{out}\cdot\frac{R_1}{R_1 + Z_1} + V_{in}\cdot\frac{Z_1}{R_1 + Z_1}\right)$$

Replace Epsilon in the first expression

$$V_{out} = -\left(V_{out}\cdot\frac{R_1}{R_1 + Z_1} + V_{in}\cdot\frac{Z_1}{R_1 + Z_1}\right)\cdot A_{OL}\cdot\frac{1}{\left(1 + \frac{s}{\omega_{pLF}}\right)\cdot\left(1 + \frac{s}{\omega_{pHF}}\right)}$$

$$\frac{V_{out}}{V_{in}} = -\frac{A_{OL}\cdot Z_1(s)\cdot\omega_{pHF}\cdot\omega_{pLF}}{R_1\cdot s^2 + Z_1(s)\cdot s^2 + R_1\cdot s\cdot\omega_{pHF} + R_1\cdot s\cdot\omega_{pLF} + R_1\cdot\omega_{pHF}\cdot\omega_{pLF} + Z_1(s)\cdot s\cdot\omega_{pHF} + Z_1(s)\cdot s\cdot\omega_{pLF} + Z_1(s)\cdot\omega_{pHF}\cdot\omega_{pLF} + A_{OL}\cdot R_1\cdot\omega_{pHF}\cdot\omega_{pLF}}$$

This is the raw transfer function accounting for the non-ideal op amp. Z1 is the feedback network impedance.

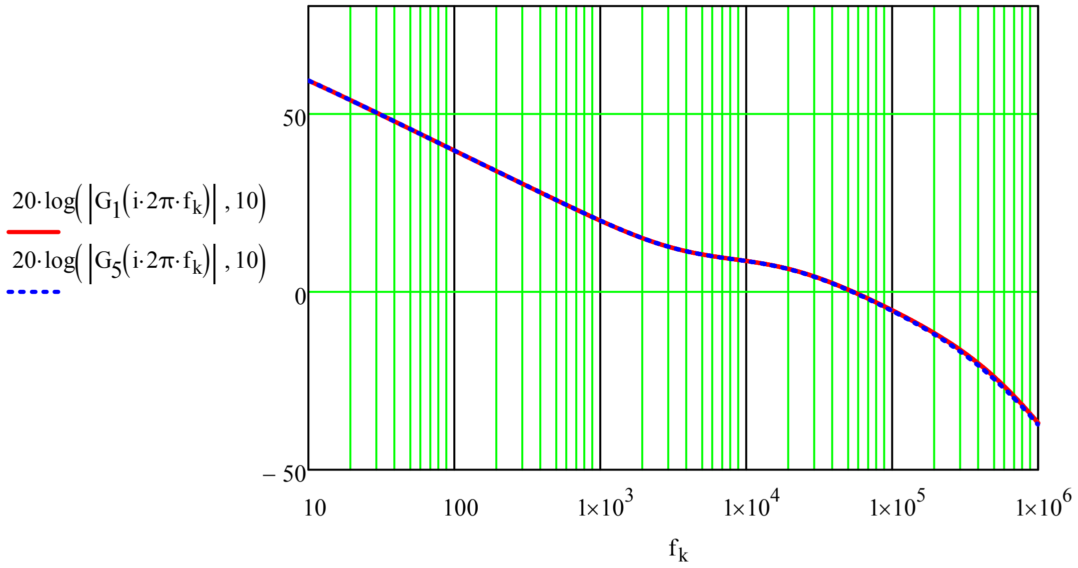
Impedance calculations

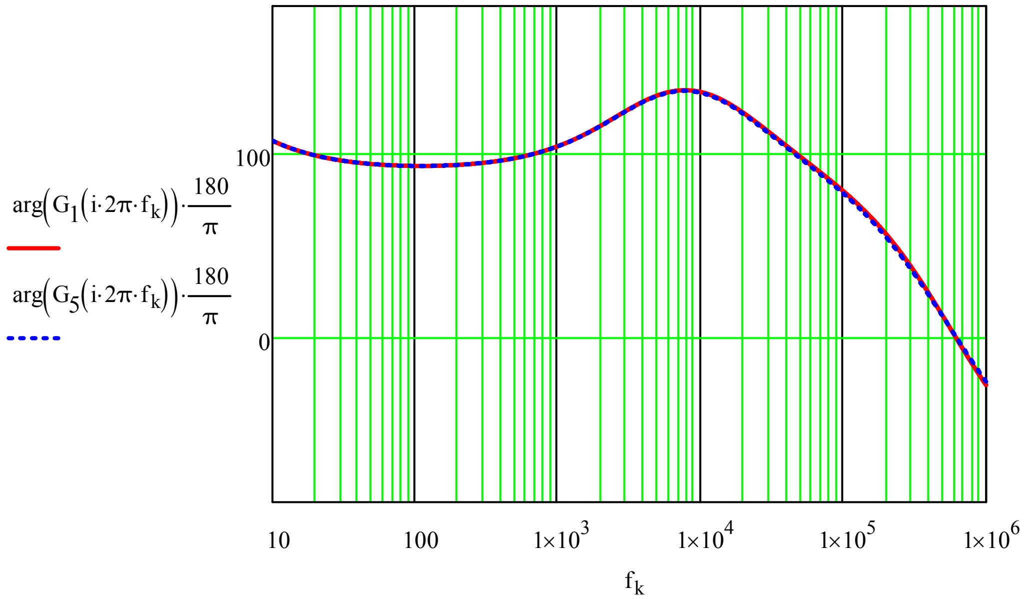
Intermediate calculations

Final approximate expression of Type 2 including AOL and the two op amp poles

$$G_5(s) := G_{t2}(s)\cdot\frac{1}{1 - \frac{G_{t2}(s)}{A_{OL}}}\cdot\frac{1}{1 + \frac{s}{\omega_{pHF} + \omega_{pLF}}}\cdot\frac{1}{1 + \frac{s}{\frac{\omega_{pHF}\cdot\omega_{pLF}}{\omega_{pHF} + \omega_{pLF}}\cdot\left(1 - \frac{A_{OL} - 1}{G_{t2}(s) - 1}\right)}}$$

Check between the raw transfer function G1 and the approximate factored version G5:



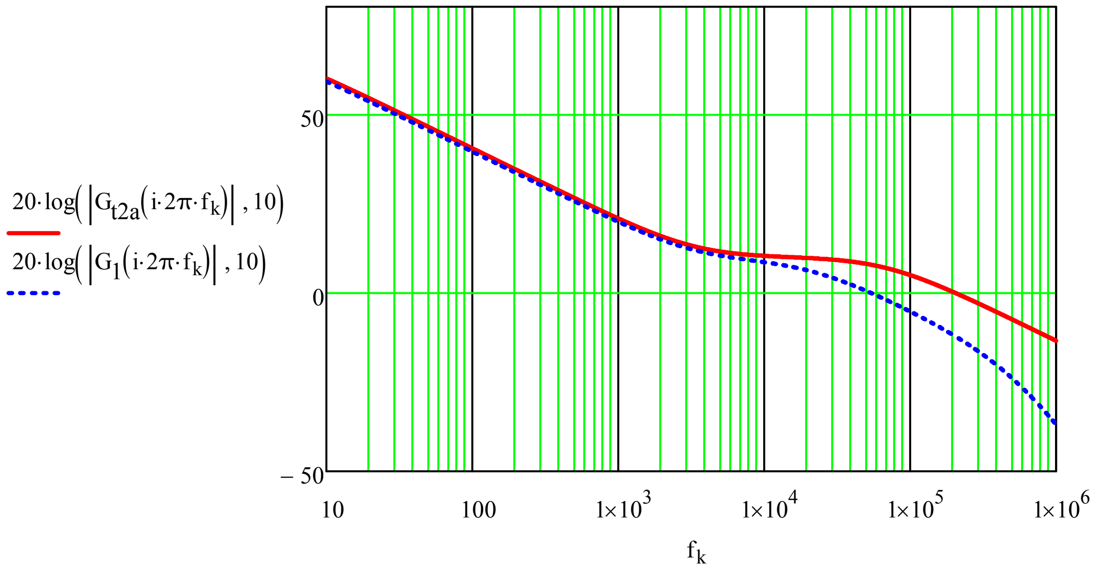


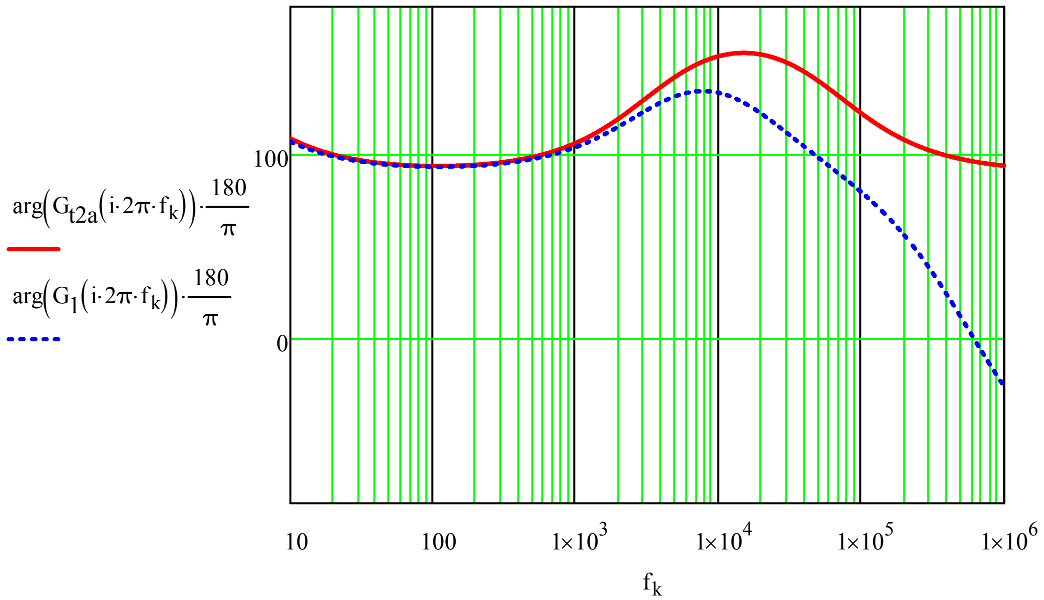
Contribution factors

Mid-band gain and phase distorsions induced by the op amp

Type 2 transfer function affected by AOL alone:

$$G_{t2a}(s) := G_{t2}(s)\cdot\frac{1}{1-\dfrac{G_{t2}(s)}{A_{OL}}}$$





Design specs: $G_{MB} = 10\ \text{dB}$ $\quad$ $Boost = 65\cdot°$

Type 2 compensator accounting for AOL only

$20 \cdot \log\left(\left|G_{t2a}(i \cdot 2\pi \cdot f_c)\right|, 10\right) = 9.992$ Obtained mid-band gain $\arg\left(G_{t2a}(i \cdot 2\pi \cdot f_c)\right) - 90° = 65.024 \cdot °$ Effective boost at crossover

Complete transfer function including op amp specs:

$20 \cdot \log\left(\left|G_1(i \cdot 2\pi \cdot f_c)\right|, 10\right) = 7.387$ Obtained mid-band gain $\arg\left(G_1(i \cdot 2\pi \cdot f_c)\right) - 90° = 37.967 \cdot °$ Effective boost at crossover

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