Input Filter Interactions with Switching Regulators

Christophe Basso – Technical Fellow
IEEE Senior Member
Course Agenda

- A Switching Regulator as a Load
- EMI Filter Impact
- An Introduction to FACTs
- Buck Converter Input/Output Impedances
- Filtering the Input Current
- Damping the Filter
- Optimum Component Selection
- A Practical Case Study
- Cascading Converters
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EMI Filter Interaction

- A $LC$ filter is inserted to prevent input line pollution

- What load does the converter offer?

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A Negative Incremental Resistance

- Assume a 100%-efficient converter
  \[ P_{\text{out}} = P_{\text{in}} \quad \rightarrow \quad I_{\text{in}}V_{\text{in}} = I_{\text{out}}V_{\text{out}} \]

- In closed-loop operation, \( P_{\text{out}} \) is constant, no link to \( V_{\text{in}} \)
  \[ I_{\text{in}}(V_{\text{in}}) = \frac{P_{\text{out}}}{V_{\text{in}}} \]

- For a constant \( P_{\text{out}} \), if \( V_{\text{in}} \) increases, \( I_{\text{in}} \) drops

The incremental input resistance is negative

\[ R_{\text{in}} = -\frac{V_{\text{in}}^2}{P_{\text{out}}} \]
A constant-power current source shows the negative resistance

- **Parameters**
  - $P_{out}=50\text{W}$

- **Diagram**
  - Bulk
  - Current: $\{Pout\}/V(\text{bulk})$
  - Infinite bandwidth

- **Small Signal DC Transfer Function**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>output impedance at $V(\text{bulk})$</td>
<td>$0.000000\text{e+000}$</td>
</tr>
<tr>
<td>$\text{vin}#\text{Input impedance}$</td>
<td>$-2.000000\text{e+002}$</td>
</tr>
<tr>
<td>Transfer function</td>
<td>$1.000000\text{e+000}$</td>
</tr>
</tbody>
</table>

Neg. resistance
A Simple $LC$ Filter

- The low-pass filter is built with $L$ and $C$ elements:

$$H(s) = H_0 \frac{1+s/\omega_z}{1+s/\omega_0 + \left(\frac{s}{\omega_0}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{r_L + R}{r_c + R}} \quad \omega_z = \frac{1}{r_c C}$$

$$Q = \frac{LC \omega_0 (r_c + R)}{L + C (r_c r_L + r_L R + r_c R)}$$

A negative resistance cancels losses: poles become imaginary

Sustained oscillations
A Negative Resistance Oscillator

- If losses are compensated, the damping ratio is zero

$$H(s) = H_0 \frac{1+s/\omega_z}{s^2 + \frac{s}{\omega_0} + 1} \quad Q = \frac{1}{2\zeta}$$

If ohmic losses are gone, the damping ratio is zero, $Q$ is infinite.

- Without precautions, instability can happen!

---

If losses are compensated, the damping ratio is zero.

$$H(s) = H_0 \frac{1+s/\omega_z}{s^2 + \frac{s}{\omega_0} + 1} \quad Q = \frac{1}{2\zeta}$$

If ohmic losses are gone, the damping ratio is zero, $Q$ is infinite.

Without precautions, instability can happen!
What is the output impedance of an $LC$ filter?

$$Z_{th}(s) = R_0 \frac{N(s)}{D(s)}$$

No dimension

$$Z_{th}(s) = \left( r_L \parallel R_{load} \right) \frac{\left( 1 + s \frac{L}{r_L} \right) \left( 1 + sr_C C_{out} \right)}{1 + s \left( \frac{L}{r_L + R_{load}} + C \left[ r_L \parallel R_{load} + r_C \right] \right) + s^2 \left( LC \frac{r_C + R_{load}}{r_L + R_{load}} \right)}$$

All losses (ohmic, iron etc.) help decreasing $Q$
At low frequency, the inductive ohmic loss dominates.
Negative Resistance at Low Frequency

- Neg. resistance exists because of feedback ($P_{out} = \text{constant}$)

Impedance measurement setup

Average model

SwitchMode Power Supplies: SPICE Simulations and Practical Designs
Christophe Basso - McGraw-Hill, 2014
The resistance is truly negative up to 200 Hz.

Well before crossover, the -180° argument is gone.
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A Filter Affects the Line Output Impedance

- The line impedance driving the converter is no longer 0

The system can be modeled by a minor loop

\[ V_{in}(s) = V_{th}(s) \frac{1}{1 + \frac{Z_{th}(s)}{Z_{in}(s)}} \]

\[ V_{th}(s) + \frac{Z_{th}}{Z_{in}} \]

"Input Filter Considerations in Design and Application of Switching Regulators", R. D. Middlebrook, IEEE Proceedings, 1976
Stability can be at stake when inserting the filter

The Nyquist criterion applies

\[ V_{in}(s) = V_{th}(s) \left( \frac{1}{1 + \frac{Z_{th}(s)}{Z_{in}(s)}} \right) \]

\[ \frac{Z_{th}(s)}{Z_{in}(s)} = -1 \]

Conditions for oscillations

\[ \left| \frac{Z_{th}(s)}{Z_{in}(s)} \right| = 1 \text{ and } \angle \frac{Z_{th}(s)}{Z_{in}(s)} = -180^\circ \]
A Filter Modifies Converter’s Dynamics

- The EMI filter output impedance makes $Z_{th}(s) \neq 0$
- The converter input voltage is no longer zero in ac analysis
It can be shown how an EMI filter affects the open-loop gain.

\[ H(s) = \frac{V_{out}(s)}{D(s)} \bigg|_{Z_{th}(s)=0} \]

Gain without the filter

\[ Z_{th}(s) \]

Filter output impedance

\[ H(s) = \frac{V_{out}(s)}{D(s)} \bigg|_{Z_{th}(s)\neq 0} = \frac{V_{out}(s)}{D(s)} \bigg|_{Z_{th}(s)=0} \]

Extra-Element correction factor

\[ 1 + \frac{Z_{th}(s)}{Z_N(s)} \]

\[ Z_{th}(s) \ll |Z_N(s)| \]

\[ 1 + \frac{Z_{th}(s)}{Z_D(s)} \]

\[ Z_{th}(s) \ll |Z_D(s)| \]
What are $Z_D$ and $Z_N$?

- $Z_D$ and $Z_N$ come from the Extra-Element Theorem, EET

$$Z_D(s) = Z_i(s)igg|_{D(s)=0}$$  Open-loop input impedance

$$Z_N(s) = Z_i(s)igg|_{\hat{v}_{out}=0}$$  Input impedance for a nulled output

$Z_D(s) = ?$  Open-loop input impedance

$Z_N(s) = ?$  Input impedance for $\hat{v}_{out} = 0$

Ideal control $\rightarrow \hat{d} \neq 0$

$\hat{v}_{out} \neq 0$  $\hat{d} = 0$  $\hat{v}_{out} = 0$

$D_0 = 35\%$  $\hat{i}_{out} \neq 0$  $\hat{i}_{out} = 0$
Consider Closed-Loop Input Impedance

- These values have already been derived

\[ Z_e(s) \text{ is the input impedance for a shorted output} \]

<table>
<thead>
<tr>
<th>Converter</th>
<th>( Z_N(s) )</th>
<th>( Z_D(s) )</th>
<th>( Z_e(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>( -\frac{R}{D^2} )</td>
<td>( \frac{R}{D^2} \left(1 + \frac{sL}{1+sRC}\right) )</td>
<td>( \frac{sL}{D^2} )</td>
</tr>
<tr>
<td>Boost</td>
<td>( -D^2 R \left(1 - \frac{sL}{D^2 R}\right) )</td>
<td>( D^2 R \left(1 + \frac{sL}{1+sRC}\right) )</td>
<td>( sL )</td>
</tr>
<tr>
<td>Buck-Boost</td>
<td>( -\frac{D^2 R}{D^2} \left(1 - \frac{sDL}{D^2 R}\right) )</td>
<td>( \frac{D^2 R}{D^2} \left(1 + \frac{sL}{1+sRC}\right) )</td>
<td>( \frac{sL}{D^2} )</td>
</tr>
</tbody>
</table>

- A converter closed-loop input impedance follows the form

\[
\frac{1}{Z_{\text{in}}(s)} = \frac{1}{Z_N(s)} \frac{T(s)}{1 + T(s)} + \frac{1}{Z_D(s)} \frac{1}{1 + T(s)}
\]

The Output Impedance is Affected

- The filter degrades the closed-loop output impedance

\[ Z_{out,CL}(s) \bigg|_{Z_{th}(s) \neq 0} = Z_{out,CL}(s) \bigg|_{Z_{th}(s) = 0} \frac{1 + \frac{Z_{th}(s)}{Z_e(s)}}{1 + \frac{Z_{th}(s)}{Z_{in,CL}(s)}} \]

- No impact if \( Z_{th}(s) \ll Z_e(s) \)

- \( Z_e(s) \) is the converter input impedance with a shorted output

\[ V_{out} = 0 \]

Short circuit
Applying Dr. Middlebrook Criteria

- Previous equations consider switching cell alone
  - Do not include the decoupling capacitor!

- If possible, move the capacitor to the filter side

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“Input Impedance and Filter Interactions Part I”, R. Ridley, ridleyengineering.com
At 1st-Order Check Crossover Points

- Plot the converter input resistance value
- Plot the filter output impedance: check for overlaps: \(|Z_{th}(s)| = |Z_{in}(s)|\)

Overlap occurs?

Watch the argument of \(\frac{Z_{th}(s)}{Z_{in}(s)}\)

\[ |Z_{out}(s)| \]
Condition for Oscillations

- Plot the ratio of magnitude $\left| \frac{Z_{th}}{Z_{in}} \right|$
- Plot the difference of arguments $\arg Z_{th} - \arg Z_{in}$

Check phase margin at 0-dB points if any

Conditions for oscillations

$$\left| \frac{Z_{th}(s)}{Z_{in}(s)} \right| = 1$$
and
$$\angle \frac{Z_{th}(s)}{Z_{in}(s)} = -180^\circ$$
Damping the filter to Build Margin

- Damping the filter can provide phase margin at 0 dB
- The phase margin in this example is a bare 12°

Avoid magnitude overlaps by working on $Z_{th}$ and $Z_{in}$!

![Graph showing phase margin and magnitude overlap](image-url)
You design the input filter together with the SMPS

\[ |Z_{th}(s)| << |Z_N(s)| \]
\[ |Z_{th}(s)| << |Z_D(s)| \]

and

\[ |Z_{th}(s)| << |Z_e(s)| \]

When a filter is installed:

Converter loop gain \( T \) is unaffected

Converter output impedance is unaffected

You design the input filter alone

\[ \left| \frac{Z_{th}(s)}{Z_{in}(s)} \right| \quad \text{and} \quad \left| \frac{Z_{th}(s)}{Z_{in}(s)} \right| \]

Don’t care about phase anymore

If damping the filter guarantees

\[ |Z_{th}(s)| << |Z_{in}(s)|_{\text{closed loop}} \]

OK

System is stable but overall performance may be altered
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**Classical Circuits Analysis Techniques**

- Apply classical Kirchhoff’s voltage and current laws
  - The expression is correct but disorganized: *high-entropy* form

\[
H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 + sC_2}{\left(\frac{1}{sC_2} + r_c\right)\left(1 + r_L + sL_1\right)}
\]

What is the transfer function?

- Apply brute force algebra

No insight: poles? Zeros? Dc gain?
FACTs describe a set of tools to quickly write transfer functions

\[
H(s) = \frac{R_1}{R_1 + r_L} \left[ 1 + s \left( \frac{L_1}{r_L + R_1} + C_2 \left( r_C + r_L \parallel R_1 \right) \right) \right] + s^2 L_1 C_2 \frac{r_C + R_1}{r_L + R_1}
\]

- Dc gain

Naturally showing gains, poles and zeros...

\[
H(s) = \left[ \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q}} \right] \left[ \frac{1}{\omega_0 Q} \right] \frac{1}{\omega_0} \frac{r_L + R_1}{L_1 + C_2 \left[ r_C r_L + R_1 \left( r_C + r_L \right) \right]}\]

This is a low-entropy expression.
Two Different Stages

- Consider dc and high-frequency states for $L$ and $C$

  - $C$
    - **Dc state**: $Z_C = \infty$ (Cap. is an open circuit)
    - **HF state**: $Z_C = 0$ (Cap. is a short circuit)

  - $L$
    - **Dc state**: $Z_L = 0$ (Inductor is a short circuit)
    - **HF state**: $Z_L = \infty$ (Inductor is an open circuit)

- Change the circuit depending on $s$

  - $s \to \infty$: $s = 0$
  - $s \to \infty$: $s = 0$
Principles of FACTs: Time Constants

- Determine time constants in two different conditions
  1. The excitation is set to zero (no excitation)
  2. The output is nulled (no response)

How do you determine a time constant?

Remove capacitor

Remove $L$ or $C$ and look into its terminals:

$R = \frac{V_T}{I_T} = R_2 \parallel R_3 + R_1$

$\tau = \left( R_2 \parallel R_3 + R_1 \right) C_1$

In your head, imagine an ohm-meter placed across $C_1$'s terminals

Excitation is set to 0
Turning the excitation off means:
- A 0-V source becomes a short circuit
- A 0-A generator is an open circuit and disappears

The inverse of the time constant in this case is a pole:

\[ \omega_p = \frac{1}{\tau} \]
A zero is the root of the equation \( f(x) = 0 \)

\[ f(x) = x^2 - 4 \]

\( f(x) = 0 \)

\( x_1 = -2 \)

\( x_2 = 2 \)

Transfer function zeros are the numerator roots

\[ N(s) = 0 \quad \rightarrow \quad s_{z_1}, s_{z_2}, \ldots \]
If the numerator is 0, then the response is theoretically 0

Complex excitation

\[ s = s_z \]

Complex response

\[ N(s_z) = 0 \]

What is happening in the box when \( s = s_z \)?

The excitation cannot reach the output: the response is nulled

\[ V_{out}(s_z) = 0 \iff \hat{v}_{out}(s_z) = 0 \]
How Does the Response Disappear?

- The signal is lost in the \textit{transformed} network

\[
Z_1(s_z) \rightarrow \infty
\]

- A series impedance becomes infinite.

\[
V_{in}(s_z)
\]

excitation

\[
s = j\omega + \sigma
\]

- A parallel impedance \textit{shorts} the path to ground

\[
V_{out}(s_z) = 0
\]

- \textit{What is a transformed} network?

\[
R_1
\]

Response $V_{out}(s_z) = 0$

\[
V_{in}(s_z)
\]

excitation

\[
s = j\omega + \sigma
\]
The Transformed Network

- Reactances are replaced by their Laplace expression

- The circuit is then observed at the zero angular frequency
Considering a Negative Angular Frequency

- For \( s = s_{z1} \), the RC impedance is a short circuit

\[
Z_1(s) = \frac{1 + sr_C C_1}{sC_1} = 0
\]

\[
Z_1(s_{z1}) = 0 \, \Omega
\]

\[
s_{z1} = -\frac{1}{r_C C_1}
\]

- For \( s = s_{z2} \), the RL impedance is infinite

\[
Z_2(s) = \frac{sL_2 R_2}{R_2 + sL_2} = 0
\]

\[
Z_2(s_{z2}) \rightarrow \infty \, \Omega
\]

\[
s_{z2} = -\frac{R_2}{L_2}
\]

Poles of the RL network become zeros of the transfer function.
Zeros by Inspection: Fastest Option!

- Identify transformed open circuits/short circuits

\[
N(s) = \left(1 + \frac{s}{\omega_{z_1}}\right) \left(1 + \frac{s}{\omega_{z_2}}\right) \left(1 + \frac{s}{\omega_{z_3}}\right)
\]

\[
V_{in}(s) \quad \text{and} \quad V_{out}(s)
\]

\[
s_{z_1} = -\frac{r_L}{L_1} \quad \omega_{z_1} = \frac{r_L}{L_1}
\]

\[
s_{z_2} = -\frac{R_2}{L_2} \quad \omega_{z_2} = \frac{R_2}{L_2}
\]

\[
s_{z_3} = -\frac{1}{r_C C_1} \quad \omega_{z_3} = \frac{1}{r_C C_1}
\]

No equations!
How would you calculate $V_{out} / V_{in}$?

1. Transform the circuit with a Thévenin generator
2. Apply impedance divider involving $C_1$
Apply Impedance Divider

- Reduce circuit complexity with Thévenin

\[ R_{th}(s) = R_1 \parallel R_2 + R_3 \]

\[ Z_1(s) = R_4 \parallel \left( r_C + \frac{1}{sC_1} \right) \]

\[ H(s) = \frac{Z_1(s)}{Z_1(s) + R_{th}(s)} \frac{R_2}{R_1 + R_2} \]

“Who you gonna call?”
How do you make use of this result?

- what is the pole/zero position?
- what affects the quasi-static gain for $s = 0$?

You can plot the ac response but it yields no insight on what drives poles and zeros!
What is the gain when $V_{in}$ is a dc voltage?

The capacitor is open circuited, read the schematic!

$$H_0 = \frac{R_2}{R_1 + R_2} \frac{R_4}{R_1 || \frac{R_2 + R_3 + R_4}{}}$$
Determine the First Time Constant

- Look at the resistance driving the storage element

1. When the excitation is turned off, $V_{in} = 0\ V$

$$\tau_1 = \left[ r_C + \left( \frac{1}{R_1} + \frac{1}{R_2 + R_3} \right) \frac{1}{R_4} \right] C_1$$

Denominator calculation
Determine the Second Time Constant

- Inspect the circuit and find the transformed short circuit

\[ V_{in} (s_z) \]

\[ V_{out} (s_z) = 0 \]

- If \( Z_1 \) is equal to 0, the output is nulled

\[ r_C + \frac{1}{s_z C_1} = 0 \]

\[ s_z = -\frac{1}{r_C C_1} \]
Assemble the Terms

- You immediately have a low-entropy form

\[
H(s) = H_0 \cdot \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}
\]

\[
H_0 = \frac{R_2}{R_1 + R_2} \cdot \frac{R_4}{R_1 \| R_2 + R_3 + R_4}
\]

\[
\omega_p = \frac{1}{\left[ r_c + (R_1 \| R_2 + R_3) \| R_4 \right] C_1}
\]

\[
\omega_z = \frac{1}{r_c C_1}
\]

- We did not write a single line of algebra!

Way cool!
Superimposing both transfer functions, matching should be perfect. If not, there is mistake.
Fractions and Dimensions

- A 1\textsuperscript{st}-order system follows the form

\[ H(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s}{b_0 + b_1 s} \]

factoring

\[ H(s) = \frac{a_0}{b_0} \left( \frac{1 + \frac{a_1}{a_0} s}{1 + \frac{b_1}{b_0} s} \right) \]

- A leading term (if any) carries the unit

\[ Z(s) = R_0 \frac{\frac{a_1}{a_0}}{1 + \frac{b_1}{b_0} s} \]

Unitless

\[ 1 + \frac{a_1}{a_0} s \]

Unitless

\[ 1 + \frac{b_1}{b_0} s \]

Unitless

\[ \frac{a_1}{a_0} \rightarrow [s] \rightarrow \tau_N \text{ time} \]

\[ \frac{b_1}{b_0} \rightarrow [s] \rightarrow \tau_D \text{ time} \]
A 2nd-order system follows the form

\[ H(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2}{\beta_0 + \beta_1 s + \beta_2 s^2} \]

Factoring \( \alpha_0 \)

\[ \text{Factoring } \beta_0 \]

The second fraction is unitless

\[ \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2} \]

Carries the unit

\[ a_1 = \frac{\alpha_1}{\alpha_0} \rightarrow [s] \rightarrow \tau_{1N} + \tau_{2N} \]

sum

\[ a_2 = \frac{\alpha_2}{\alpha_0} \rightarrow \left[ s^2 \right] \rightarrow \tau_{1N} \tau_{2N} \text{ or } \tau_{2N} \tau_{1N} \]

product

\[ b_1 = \frac{\beta_1}{\beta_0} \rightarrow [s] \rightarrow \tau_{1D} + \tau_{2D} \]

\[ b_2 = \frac{\beta_2}{\beta_0} \rightarrow \left[ s^2 \right] \rightarrow \tau_{1D} \tau_{2D} \text{ or } \tau_{2D} \tau_{1D} \]

reactance 1 reactance 2
Alternating the Reactance States

- In a 1\textsuperscript{st}-order circuit, there is one reactance
  - it is either in a high-frequency state or in a dc state

- In a 2\textsuperscript{nd}-order circuit, there are two reactances
  - we can consider individual states

\[ s = 0 \quad R \quad \tau^2_1 \quad L \quad C \quad H_0 \quad \begin{array}{c} \tau^2_1 \quad L \\ C \quad \end{array} \quad s \rightarrow \infty \quad R \quad L \quad C \quad H_\infty \]
Introducing the Notation

- Set one reactance into its high-frequency state

- Reactance 1 is in its high-frequency state

- What resistance drives reactance 2?

- Reactance 2 is in its high-frequency state

- What resistance drives reactance 1?

- There is redundancy: pick the simplest result

\[ b_2 = \tau_1 \tau_2^1 \quad \leftrightarrow \quad b_2 = \tau_2 \tau_1^2 \]
Example with Capacitors

- Assume the following 2-capacitor circuit

\[ V_{in}(s) \quad R_1 \quad r_C \quad C_1 \quad V_{out}(s) \quad s = 0 \quad s = 0 \quad H_0 = 1 \]

- Determine the two time constants while \( V_{in} \) is 0 V

\[ \tau_1 = C_1 (r_C + R_1) \quad b_1 = \tau_1 + \tau_2 \]

\[ \tau_2 = C_2 R_1 \quad b_1 = C_1 (r_C + R_1) + C_2 R_1 \]
Determining the Higher-Order Term

- Place $C_1$ in its high-frequency state and look into $C_2$

\[ R_1 \quad r_C \quad C_1 \quad C_2 \]

\[ V_{in} = 0 \text{ V} \]

\[ \tau_2 = (R_1 \parallel r_C) C_2 \]

\[ b_2 = \tau_1 \tau_2^1 = C_1 (r_C + R_1) C_2 (R_1 \parallel r_C) \]

- Place $C_2$ in its high-frequency state and look into $C_1$

\[ R_1 \quad r_C \quad C_1 \quad C_2 \]

\[ V_{in} = 0 \text{ V} \]

\[ \tau_1^2 = r_C C_1 \]

\[ b_2 = \tau_2 \tau_1^2 = C_2 R_1 C_1 r_C \]
The denominator can be assembled

\[ D(s) = 1 + b_1 s + b_2 s^2 = 1 + \left[ C_1 (r_C + R_1) + C_2 R_1 \right] s + C_2 R_1 C_1 r_C s^2 \]

Is there a zero in this network?

If \( Z_1 \) becomes a transformed short, the response disappears
Final Expression and Conclusion

- Gather the pieces to form the transfer function

\[
H(s) = \frac{1 + sr_C C_1}{1 + [C_1 (r_C + R_1) + C_2 R_1] s + C_2 R_1 C_1 r_C s^2}
\]

\[
H(s) = H_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}
\]

- This expression was determined in a flashing time

- We did not use KVL or KCL: inspection is easy

No algebra!


Linear Circuit Transfer Function – A Tutorial Introduction to Fast Analytical Techniques – Christophe Basso – Wiley & Sons 2016
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Replace the diode and the switch by the PWM switch.

Buck Input Impedance

- Inductance LoL lets you sweep the input to have $Z_{in}$

- In this mode, $\hat{d}$ is equal to zero

\[
\begin{align*}
\frac{V_{in}(s)}{I_{in}(s)} & \bigg|_{\hat{d}=0} \\
& \text{Source B2 and B1 are zero} \\
& \text{Node } p \text{ is ground} \\
& V(a, p) = V_{in} \\
& \text{Simplify schematic} \\
& \text{Check ac response} \\
& \text{Input impedance}
\end{align*}
\]
Simplifying and Rearranging is Key

- Install the dc transformer to obtain $Z_{in}$

$$
Z_{in}(s) = \frac{V_T(s)}{I_{in}(s)}
$$

- Reflect elements to the primary side

$\text{Dc input resistance}$
Start with $s = 0$ – Draw Circuit in dc

- Short the inductor, open the capacitor

\[ R_0 = \frac{r_L + R_{load}}{D_0^2} \]

- For the time constants, suppress the excitation, $I_T = 0$

\[ \tau_2 = C_2 D_0^2 \left( \frac{r_C + R_{Load}}{D_0^2} \right) \]

\[ \tau_1 = \frac{L_1}{D_0^2 \cdot \infty} \]

Input impedance
Higher Order Coefficients

- Avoid indeterminacy with $\tau_1$: use $\tau_2$ instead
- Determine $\tau_1^2$

\[
\tau_1^2 = \frac{L_1}{D_0^2 \cdot \infty}
\]

\[
b_2 = \tau_2 \tau_1^2 = C_2 D_0^2 \left( \frac{r_C + R_{Load}}{D_0^2} \right) \frac{L_1}{D_0^2 \cdot \infty} = 0
\]

\[
D(s) = 1 + \left[ C_2 D_0^2 \left( \frac{r_C + R_{Load}}{D_0^2} \right) + \frac{L_1}{D_0^2 \cdot \infty} \right] s = 1 + sC_2 \left( r_C + R_{Load} \right)
\]

Input impedance
The Numerator is of 2\textsuperscript{nd}-Order Type

- Null the response across the current source

\[ V_T = 0 \]

\[ I_T \]

\[ N(s) = 1 + s \left( \frac{L_1}{r_L + R_{\text{load}}} \right) + C_2 \left[ \left( \frac{r_L}{r_L + R_{\text{load}}} \right) + r_C \right] \] + \[ s^2 \left( \frac{L_1 C_2}{r_L + R_{\text{load}}} \right) \]

Use Fast Analytical Circuits Techniques!
The transfer function dimension is ohm

\[ Z_{in,OL}(s) = R_0 \frac{1 + \frac{s}{\omega_0 Q} + \left( \frac{s}{\omega_0} \right)^2}{1 + \frac{s}{\omega_p}} \]

\[ Q = \frac{L_1 C_2 \omega_0 (r_C + R_{load})}{L_1 + C_2 \left( r_L r_C + r_L R_{load} + r_C R_{load} \right)} \]

\[ \omega_p = \frac{1}{\sqrt{L_1 C_2} \sqrt{r_L + R_{load}} \sqrt{r_C + R_{load}}} \]

\[ R_0 = \frac{r_L + R_{load}}{D_0^2} \]

\[ \omega_p = \frac{1}{(r_C + R_{Load}) C_2} \]

\[ Z_{in,OL}(f) \]

\[ |Z_{in,OL}(f)| \]

\[ \angle Z_{in,OL}(f) \]

\[ 29.2 \text{ dBΩ} \]

\[ 0 \text{ dBΩ} \]
We want the input impedance once the loop is closed.

\[ H(s) \]

\[ G_{PWM} \]

\[ Z_{in}(s) \]

\[ V_{err}(s) = V_{out}(s)G(s) \]

\[ V_{out}(s) \]

\[ G(s) \]
Stabilize the Buck with a Type 3

- A 10-kHz crossover is selected for the compensation

![Circuit Diagram]

**Parameters**

- \( V_{out} = 5 \)
- \( P_{out} = 50 \)
- \( R_{load} = V_{out}/2/P_{out} \)
- \( R_{upper} = 10k \)
- \( A_{OL} = 1T \)
- \( V_{p} = 2 \)
- \( f_{c} = 10k \)
- \( V_{in} = 20 \)
- \( G_{fc} = -22 \)
- \( P_{M} = 70 \)
- \( P_{S} = -119 \)
- \( \text{boost} = PM - PS - 90 \)

\[ G = 10^{\left(-G_{fc}/20\right)} \]

\[ \pi = 3.14159 \]

- \( f_{z1} = 540 \)
- \( f_{z2} = 230 \)
- \( f_{p1} = 6.8k \)
- \( f_{p2} = f_{p1} \)

\[ C_{1} = 1/(2\pi f_{z1} f_{p1} R_{2}) \]

\[ C_{2} = C_{1}/(C_{1} R_{2} 2\pi f_{p1} - 1) \]

\[ C_{3} = (f_{p2} - f_{z2})/(2\pi R_{upper} f_{p2} f_{z2}) \]

\[ R_{3} = R_{upper} f_{z2}/(f_{p2} - f_{z2}) \]

\[ a = \sqrt{(f_{c}^2/f_{p1}^2 + 1)} \]

\[ b = \sqrt{(f_{c}^2/f_{p2}^2 + 1)} \]

\[ c = \sqrt{(f_{z1}^2/f_{c}^2 + 1)} \]

\[ d = \sqrt{(f_{z2}^2/f_{c}^2 + 1)} \]

\[ R_{2} = ((a b/c d)/(f_{p1} - f_{z1})) R_{upper} G_{p1} \]

\[ G_{0} = ((R_{2} C_{1})/(R_{upper} (C_{1} + C_{2}))) \]

\[ f_{c} = 10 \text{ kHz} \]

\[ \varphi_{m} = 60^\circ \]

\[ T(f) = H(s) G(s) \]

\[ \angle T(f) \]

Public Information
Christophe Basso – Input Filter Interactions
Use Large-Signal Model for Reference

- Use the PWM switch to check the response

![Circuit Diagram]

Setup for input impedance measurement.

\[ H \]

\[ d(f) \]

\[ V_{err}(f) \]

\[ G(f) \]

compensator

Public Information
Christophe Basso – Input Filter Interactions
Loop Gain Decreases as $f$ Increases

- Input impedance is negative at low frequencies only

![Graph showing the absolute value and phase of $Z_{in,CL}(f)$]

$|Z_{in,CL}(f)|$ and $\angle Z_{in,CL}(f)$ plotted over frequency. Simulation results indicate $\angle Z_{in}(f) > 0$ and $\angle Z_{in}(f) = -180^\circ$. 

$dB \Omega$ vs frequency plot from 1 to 100k Hz.
Why Does $Z_{in}$ Become Positive?

- The below expression only holds if $P_{out}$ is truly constant

$$R_{in} = -\frac{V_{in}^2}{P_{out}}$$

Infinite input voltage rejection

- It is true at dc, where loop gain is very high

$$A_{SC,OL}(s) = D \frac{R_{load}}{R_{load} + r_L} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$

Audio susceptibility

$$A_{SC,CL}(s) = \frac{A_{SC,OL}(s)}{1 + T(s)}$$

Loop gain quickly drops as $f$ increases

$A_{SC}$, audio susceptibility
Determining the Closed-Loop Impedance

- Update the large-signal model with linear sources

- \( B_2 \) needs to be linearized as \( d \) and \( V_{in} \) now include ac

\[
(V_{in} + \hat{v}_{in})(D_0 + \hat{d}) \approx \hat{d}V_{in} + \hat{v}_{in}D_0
\]
Simplified Circuit Helps Analysis

- Final closed-loop schematic to perform analysis

- Sanity check is mandatory!

$V_{in}(s) \rightarrow I_C(s) \rightarrow Z_{RLC}(s) \rightarrow H_p(s)$

$G_{PWM}(s) \rightarrow V_{err}(s) \rightarrow G(s)(s) \rightarrow V_{out}(s)$

$Z_{RLC}(s)$ is the filter input impedance
Derive Equations for Individual Variables

- The duty ratio is linked to $V_{out}$ by the error amplifier $G$
  \[ D(s) = G(s)G_{PWM}V_{out}(s) \]

- From the previous slide, we have
  \[ V_{out}(s) = \left[D_0 V_{in}(s) + D(s)V_{in}\right] \frac{H(s)}{V_{in}} \]
  \[ D(s) = \frac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)} \]

- Current in terminal c is $B_4$ applied to the $RLC$ network
  \[ V_{in}(s) D_0 + V_{in} \frac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}{V_{in} - G_{PWM} V_{in} G(s) H(s)} \]
  \[ I_C(s) = \frac{V_{in}(s) D_0 + V_{in} \frac{D_0 \cdot G_{PWM} G(s) H(s) V_{in}(s)}}{Z_{RLC}(s)} \]
The input current depends on the two input sources:

\[ I_{in}(s) = I_{out}D(s) + I_C(s)D_0 \]

Buck dc output current \[ \text{Static duty ratio} \]

Substitute \( D(s) \) and \( I_C(s) \) previously obtained

\[ I_{in}(s) = I_{out} \frac{D_0 \cdot G_{PWM} G(s)H(s)V_{in}(s)}{V_{in} - G_{PWM}V_{in}G(s)H(s)} + D_0 \frac{D_0 + V_{in}}{V_{in} - G_{PWM}V_{in}G(s)H(s)} \frac{D_0 \cdot G_{PWM} G(s)H(s)V_{in}(s)}{V_{in} - G_{PWM}V_{in}G(s)H(s)} \frac{Z_{RLC}(s)}{Z_{RLC}(s)} \]

Factor \[ T(s) \]

Rearrange

\[ \frac{1}{V_{in}(s)} = \frac{I_{in}(s)}{V_{in}(s)} = \frac{D_0I_{out}}{V_{in}} \frac{G_{PWM}G(s)H(s)}{1 - G_{PWM}G(s)H(s)} + \frac{D_0^2}{Z_{RLC}(s)} \left( 1 + \frac{G_{PWM}G(s)H(s)}{1 - G_{PWM}G(s)H(s)} \right) \]
Final Expression for $Z_{in}$ in Closed Loop

- Final expression involves two contributors

$$\frac{1}{Z_{in}(s)} = \frac{D_0^2}{R_{load}} \frac{T(s)}{1-T(s)} + \frac{D_0^2}{Z_{RLC}(s)} \left( \frac{1}{1-T(s)} \right)$$

- In dc or low frequency, $T(s)$ is $>> 1$

$$\frac{1}{Z_{in}(s)} \approx \frac{D_0^2}{R_{load}} \frac{T(s)}{1-T(s)} \approx \frac{D_0^2}{R_{load}} \quad \rightarrow \quad Z_{in}(s) \approx -\frac{R_{load}}{D_0^2} \quad s \rightarrow 0$$

- As $s$ exceeds $f_c$ and increases, $T(s)$ is $<< 1$

$$\frac{1}{Z_{in}(s)} \approx \frac{D_0^2}{Z_{RLC}(s)} \left( \frac{1}{1-T(s)} \right) \approx \frac{D_0^2}{Z_{RLC}(s)} \quad \approx 1 \quad \rightarrow \quad Z_{in}(s) \approx \frac{Z_{RLC}(s)}{D_0^2}$$

No gain means open-loop operation
Same as $D(s) = 0$

$$Z_{RLC}(s) = \frac{1 + \frac{s}{\omega_0 Q} + \left( \frac{s}{\omega_0} \right)^2}{s^2\left( \frac{r_c + R_{load}}{C_1} \right)}$$
Plot the Closed-Loop Input Impedance

- Magnitude becomes open-loop plot in high-frequency

- Argument meets open-loop plot in high-frequency
  - Negative argument occurs only at low-frequency
Open-Loop Output Impedance

- What is the buck converter output impedance?

\[ Z_{out}(s) = \frac{V_{out}(s)}{I_{out}(s)} \]

- Consider parasitic elements for \( L \) and \( C \)

\[ Z_{out}(s) = \frac{V_{out}(s)}{I_{out}(s)} \]
Buck Output Impedance

Let's find the term $R_0$ in dc: open caps, short inductors

$$R_0 = \frac{V_T}{I_T} = r_L \| R_{\text{load}}$$

The zeros cancel the response

$$sL_1 + r_L = 0 \quad r_L \left( \frac{sL}{r_L} + 1 \right) = 0$$
$$\omega_{z_1} = \frac{r_L}{L_1}$$

$$\frac{1}{sC_2} + r_C = 0 \quad 1 + sr_C \frac{C_2}{s} = 0$$
$$\omega_{z_2} = \frac{1}{r_C C_2}$$

$$N(s) = \left( 1 + s \frac{L_1}{r_L} \right) \left( 1 + sr_C C_2 \right)$$
Low-Frequency Time Constants

- All elements are in their dc state
- Look at $R$ driving $L$ then $R$ driving $C$

\[ R = r_L + R_{load} \]

\[ R = (r_L \parallel R_{load}) + r_C \]

\[ b_1 = \frac{L_1}{r_L + R_{load}} + C_2 \left[ (r_L \parallel R_{load}) + r_C \right] \]
Set $L_1$ in high frequency state and look at $R$ driving $C_2$

$$\tau_2^1 = C_2 \left( r_c + R_{load} \right)$$

$$\tau_2^2 = \frac{L_1}{r_L + R_{load} \parallel r_C}$$

$$b_2 = \frac{L_1}{r_L + R_{load}} C_2 \left( r_c + R_{load} \right) = C_2 \left[ \left( r_L \parallel R_{load} \right) + r_C \right] \frac{L_1}{r_L + R_{load} \parallel r_C}$$

$$b_2 = \tau_1 \tau_2^1 \iff \text{redundancy} \iff b_2 = \tau_2 \tau_1^2$$
Final Expression for $Z_{OUT}$

- We have our denominator!

$$D(s) = 1 + s \left( \frac{L_1}{r_L + R_{load}} + C_2 \left( (r_L \ || \ R_{load}) + r_C \right) \right) + s^2 \left( L_1 C_2 \frac{r_C + R_{load}}{r_L + R_{load}} \right)$$

- The complete transfer function is now:

$$Z_{out,OL}(s) = \left( r_L \ || \ R_{load} \right) \frac{\left( 1 + s \frac{L_1}{r_L} \right) \left( 1 + s r_C C_2 \right)}{1 + s \left( \frac{L_1}{r_L + R_{load}} + C_2 \left( (r_L \ || \ R_{load}) + r_C \right) \right) + s^2 \left( L_1 C_2 \frac{r_C + R_{load}}{r_L + R_{load}} \right)}$$

- This is the open-loop output impedance
A Closed-Loop System

- The converter fights output perturbations

\[ V_{ref}(s) \rightarrow G(s) \rightarrow H(s) \rightarrow V_{out}(s) \]

- Because the system is linear, superposition applies

\[
V_{out1}(s) = V_{ref}(s) \cdot \frac{T_{OL}(s)}{1 + T_{OL}(s)} \quad V_{out2}(s) = I_{out}(s)Z_{out,OL}(s) - V_{out}(s)T_{OL}(s)
\]

\[
V_{out}(s) = V_{out1}(s) + V_{out2}(s) = V_{ref}(s) \cdot \frac{T_{OL}(s)}{1 + T_{OL}(s)} - I_{out}(s) \cdot \frac{Z_{out,OL}(s)}{1 + T_{OL}(s)}
\]

- The loop gain affects the final expression

\[ Z_{out,CL} \]
Loop Gain Impact on the Impedance

- Plots superimpose as loop gain approaches 0

- Having gain at $f_0$ is important to damp the filter

$$
|Z_{out,CL}(f)|
$$

$$
|Z_{out,OL}(f)|
$$

Peaking is gone in closed-loop

$$
20 \cdot \log \left( \left| \frac{Z_{outCL}(i \cdot 2\pi f_k)}{1\Omega} \right| \right)
$$

$$
20 \cdot \log \left( \left| \frac{Z_{outOL}(i \cdot 2\pi f_k)}{1\Omega} \right| \right)
$$
Course Agenda

- A Switching Regulator as a Load
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A Design Example with a Buck

Assume the following 5-V/50-W buck converter

Specifications are less than 15 mA peak of input ripple
What is the Necessary Attenuation?

- Use SPICE to analyze the input current signature

\[ I_{in,\text{rms}} = 5.4 \, \text{A} \]

\[ 11.13 \, \text{A} \]

\[ i_{in}(t) \]

\[ I_{ac} = \sqrt{I_{in,\text{rms}}^2 - I_{dc}^2} = \sqrt{5.4^2 - 2.85^2} \approx 4.6 \, \text{A} \]

Current in the capacitor

- FOUR 100kHz I(V4)

\[ I_1 = 4.94 \, \text{A peak} \]

\[ I_{in} < 15 \, \text{mA peak} \]

Attenuate by 3m or 50 dB
Where do you Place the Double Pole?

- Insert a $LC$ filter to attenuate the pulsating current

$$\frac{I_{in}}{I_{out}} \approx \left(\frac{\omega_0}{\omega}\right)^2$$

- Position $f_0$ to provide a 50-dB attenuation at 100 kHz

$$f_0 = \sqrt{A_{\text{filter}}} \cdot f_{SW} = \sqrt{3m} \times 100k \approx 17 \text{ kHz}$$
Select the Filter Elements

- The resonant frequency lets us chose $L$ and $C$

$$LC = \frac{1}{4\pi^2 f_0^2} = 8.34 \times 10^{-10} \text{ s}^2$$

- Component selection depends on volume, cost etc.
  - the capacitor sees the buck ac input current
  - the inductor ripple current is small, consider dc only

- $L = 22 \ \mu\text{H}$
  - $I_{in} < 3 \ \text{A}$

- $4 \times 10 \ \mu\text{F}$
  - $I_{tot,rms} > 40 \ \text{A}$

- Kemet F series
  - 7447714220 Würth

- Public Information
  - Christophe Basso – Input Filter Interactions
We can show that the complete attenuation is

\[
\frac{|I_{in}|}{|I_{out}|} = \sqrt{\frac{r_C^2 + \frac{1}{(C_1 \omega)^2}}{(r_C + r_L)^2 + \frac{1}{(\omega C_1)^2} - \frac{2L_1}{C_1} + (L_1 \omega)^2}}
\]

Check if attenuation is ok with selected components

\[r_C = \frac{5 \text{ m}\Omega}{4} \approx 1.3 \text{ m}\Omega\]
\[r_L = 50 \text{ m}\Omega\]

\[|I_{in}| = -50.8 \text{ dB at } 100 \text{ kHz}\]
The addition of the filter affects the converters

\[ H(s) = \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2} \]

Buck control-to-output TF without filter.

The small-signal model needs an update.
The converter becomes a 4\textsuperscript{th}-order system

Simplify the circuit before solving the function
You can reuse previously-determined transfer functions

\[ V_{in}(s) = Z_{th}(s) D(s) I_{out} \]
\[ I_{C}(s) = I_{C}(s) D_0 \]
\[ V_{out}(s) = H_p(s) \]

Small-signal model

Apply KVL and KCL to obtain the new expression
Determine and Substitute Variables

- Determine the input voltage expression \( V_{\text{in}}(s) \)
  \[ V_{\text{in}}(s) = -Z_{\text{th}}(s)\left[I_{\text{out}}D(s) + I_{C}(s)D_0\right] \]

- Terminal c current is the voltage \( V_{(c)} \) divided by \( Z_{RLC}(s) \)
  \[ I_{C}(s) = \frac{V_{\text{in}}(s)D_0 + D(s)V_{\text{in}}}{Z_{RLC}(s)} \]
  \[ V_{\text{in}}(s) = -Z_{\text{th}}(s)\left[I_{\text{out}}D(s) + \frac{V_{\text{in}}(s)D_0 + D(s)V_{\text{in}}}{Z_{RLC}(s)}D_0\right] \]

  \[ V_{\text{in}}(s) = \frac{D(s)Z_{RLC}(s)Z_{\text{th}}(s)I_{\text{out}} + D_0V_{\text{in}}D(s)Z_{\text{th}}(s)}{Z_{\text{th}}(s)D_0^2 + Z_{RLC}(s)} \]

- The output voltage involves the \( RLC \) transmittance \( H_p \)
  \[ V_{\text{out}}(s) = \left[V_{\text{in}}(s)D_0 + D(s)V_{\text{in}}\right]H_p(s) \]
Final Expression is Complicated

- Rearrange the final transfer function

\[
\frac{V_{out}(s)}{D(s)} = \frac{R_{load}V_{in}}{R_{load} + r_L} \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

Classical buck, no filter

\[
Z_{th}(s) = r_{Lf} \frac{(1 + sr_{Cf}C_f) \left(1 + s \frac{L_f}{r_{Lf}}\right)}{1 + sC_f \left(r_{Cf} + r_{Lf}\right) + s^2L_fC_f}
\]

- Filter output impedance impacts the transfer function

\[
Q_f = \frac{1}{\sqrt{\frac{L_f}{C_f} + \frac{1}{r_{Cf} + r_{Lf}}}} \quad \omega_{0f} = \frac{1}{\sqrt{L_fC_f}}
\]

\[
\omega_{z1} = \frac{1}{r_{Cf}C_f} \quad \omega_{z2} = \frac{r_{Lf}}{L_f}
\]
Plots Show a Distorted Transfer Function

- Mathcad and SPICE match each other quite well

![Graph showing transfer function](image)

- The notch occurs because $V_{in}$ drops at resonance
The Extra Element Theorem at Work

What is the EET principle?

- Identify an element whose presence complicates the analysis
- Calculate the transfer function with that element removed
- Apply a correction factor $k$ to the transfer function: voilà!

\[ H(s) = \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_0Q} + \left(\frac{s}{\omega_0}\right)^2} \]

Correction factor

\[ H(s) = H(s) \bigg|_{Z_{th}=0} \cdot k \]
Determining the Correction Factor

- The correction factor requires two terms:
  1. The converter input impedance obtained in open-loop
  2. The converter input impedance obtained for $V_{out}(s) = 0$

- The open-loop impedance has already been derived

$$Z_D(s) = \frac{R_{load} + r_L}{D_0^2} \frac{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}{1 + \frac{s}{\omega_p}}$$
The Null Propagates in the Circuit

- Despite excitation with $I_T$, there is no ac response
- This is the principle behind nulling the response

$$V_{in}(s) = 0$$

0 A in the load and 0 V at $r_L - r_C$ implies 0 V across $L_1$

$$V_{in}(s)D_0 + D(s)V_{in} = 0 \quad \Rightarrow \quad D(s) = -D_0 \frac{V_{in}(s)}{V_{in}}$$
Expression for $Z_N$ Comes Easily

- Substitute/rearrange test and voltage expressions

\[ I_T(s) = I_{out}D(s) + I_C(s)D_0 \]

\[ D(s) = -D_0 \frac{V_{in}(s)}{V_{in}} \]

\[ \frac{I_T(s)}{V_T(s)} = -\frac{I_{out}D_0}{V_{in}} \rightarrow \frac{V_{out}}{D_0} \]

- The input impedance $Z_N$ for a nulled response is:

\[ Z_N(s) = -\frac{R_{load}}{D_0^2} \rightarrow \text{Nulled response implies infinite input rejection} \]

It is the incremental input resistance seen before

- We already had this with the SPICE simulation

**Public Information**
Christophe Basso – Input Filter Interactions

**Parameters**

- **Pout=50W**

**Diagram**

For a buck

\[ R_0 = -\frac{V_{in}^2}{P_{out}} = -\frac{\left(\frac{V_{out}}{D_0}\right)^2}{\frac{V_{out}^2}{R_{load}}} = -\frac{R_{load}}{D_0^2} \]
The final expression includes the filter impact

\[
\frac{V_{out}(s)}{D(s)} = V_{in} \frac{R_{load}}{R_{load} + r_L} \left( 1 + \frac{s}{\omega_z} + \left( \frac{s}{\omega_0} \right)^2 \right) \left[ 1 + \frac{Z_{th}(s)}{\frac{R_{load}}{D_0^2}} \frac{1}{1 + \frac{Z_{th}(s)}{\frac{R_{load} + r_L}{\omega_0 Q} + \left( \frac{s}{\omega_0} \right)^2}} \right]
\]

Comparison between EET and full formula

Argument difference

Magnitude difference
What is the Impact on Stability?

- When adding the filter, the loop gain is awfully ugly!

- Filter damping is necessary:
  \[
  |Z_{th}(s)| < |Z_N(s)|
  \]
  \[
  |Z_{th}(s)| < |Z_D(s)|
  \]
The Output Impedance is Affected

- Closed-loop output impedance peaks as filter resonates

\[ Z_{out}(s) = Z_{out}(s) \bigg|_{Z_{in}(s)=0} + \frac{Z_{th}(s)}{1 + \frac{Z_{th}(s)}{Z_{in,CL}(s)}} \]

Filter damping is necessary: \[ |Z_{th}(s)| < |Z_e(s)| \]
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Check Filter Output Impedance

- Plot shows that design inequalities are not respected

\[ |Z_{th}(s)| << |Z_N(s)| \]
\[ |Z_{th}(s)| << |Z_D(s)| \]

\[ Q_f = 14.4 \]
\[ |Z_{th}(f)| \]

Gain and phase distortion of \( k \) must be minimized
Filter Peaking Also Affects $Z_{out}$

- Inequality for the closed-loop $Z_{out}$ is not respected

$$Q_f = 14.4$$

Input impedance with shorted output

$$20 \cdot \log \left( \frac{\left| Z_{th}(i \cdot 2\pi f_k) \right|}{1 \Omega} \right)$$

$$20 \cdot \log \left( \frac{\left| Z_e(i \cdot 2\pi f_k) \right|}{1 \Omega} \right)$$

- Filter damping must ensure $\left| Z_{th}(s) \right| << \left| Z_e(s) \right|$
Look at the Correction Factor $k$

- **Sweep** $Q_f$ for the least phase and gain deviation

\[
k(s) = \frac{1 - \frac{Z_{th}(s)}{R_{load}D_0^2}}{1 + \frac{1}{Z_{th}(s)} + \left(\frac{s}{\omega_p}\right)^2}
\]

\[
Z_{th}(s) = R_0 \cdot \frac{\left(1 + \frac{s}{\omega_i} \right) \left(1 + \frac{s}{\omega_z} \right)}{1 + \frac{s}{\omega_0Q_f} + \left(\frac{s}{\omega_0}\right)^2}
\]

\[
Q_f = \sqrt{\frac{L_f}{C_f r_{cf} + r_{Lf}}}
\]
Determine Maximum Filter Peaking

- Determine maximum peaking with selected $Q_f$

$$|Z_{th}|_{MAX} = \frac{R_0 Q_f}{\omega_z} \sqrt{\left(\frac{\omega_0^2}{\omega_z} + \frac{1}{\omega_z^2}\right)\left(\frac{\omega_0^2}{\omega_z} + \frac{1}{\omega_z^2}\right)}$$

<table>
<thead>
<tr>
<th>Circuit Components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
<td>50 mΩ</td>
</tr>
<tr>
<td>$Q_f$</td>
<td>14.4</td>
</tr>
<tr>
<td>$\omega_z$</td>
<td>22 μH</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>1.3 mΩ</td>
</tr>
<tr>
<td>$Z_{th}$</td>
<td>10.7 Ω</td>
</tr>
<tr>
<td>$Z_{th,MAX}$</td>
<td>0.7 Ω</td>
</tr>
</tbody>
</table>

- What damping elements will reduce $|Z_{th}|_{MAX}$ to 0.7 Ω?
Available Damping Techniques

- Damping means increasing the loss per cycle
  - Increase power dissipation

**Parallel Damping**

- $r_{Cf} - C_f$ parallel damping: $R_{damp}$ dissipates power
- $r_{Lf} - L_f$ parallel damping: $R_{damp}$ adds a zero and alters filter
Adding a Blocking Capacitor

- The series capacitor blocks the dc component
- Literature recommends \( C_{damp} = 10 \times C_f \)

- \( C_{damp} \) can be extremely bulky
- In ac \( R_{damp} \) dissipation can be an issue

\[
C_f = 40 \, \mu F
\]

\[
C_{damp} = 400 \, \mu F
\]
Analyzing the Filter Output Impedance

Three storage elements with independent state variables: 3rd order system.

\[
D(s) = 1 + b_1 s + a_2 s^2 + a_3 s^3
\]
Start with Dc Analysis \( s = 0 \)

\[ R_0 = r_L \parallel R_{inf} \]

\( R_{inf} \) is the current source output resistance and ensures a dc path when \( I_T = 0 \)
Determine Time Constants

Turn excitation off, current source is set to 0 A

\[ \tau_3 = C_3 \left( r_C + r_L \ || \ R_{\text{inf}} \right) \]
\[ \tau_2 = C_{\text{damp}} \left( R_{\text{damp}} + r_L \ || \ R_{\text{inf}} \right) \]
Assemble Time Constants for $b_1$

$$\tau_1 = \frac{L_1}{R_{\text{inf}} + r_L}$$

$$\bar{D}(s) = 1 + (\tau_1 + \tau_2 + \tau_3)s + (\tau_1 \tau_2^1 + \tau_1 \tau_3^1 + \tau_2 \tau_3^2)s^2 + (\tau_1 \tau_2 \tau_3^{12})s^3$$

$$b_1 = \tau_1 + \tau_2 + \tau_3 = \frac{L_1}{R_{\text{inf}} + r_L} + C_{\text{damp}} \left( R_{\text{damp}} + r_L \parallel R_{\text{inf}} \right) + C_3 \left( r_C + r_L \parallel R_{\text{inf}} \right)$$
Second-Order Time Constants

\[ D(s) = 1 + \left( \tau_1 + \tau_2 + \tau_3 \right) s + \left( \tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3 \right) s^2 + \left( \tau_1 \tau_2 \tau_3 \right) s^3 \]

\[ \tau_2 = C_{damp} \left( R_{damp} + R_{\text{inf}} \right) \]

\[ \tau_3 = C_3 \left( r_C + R_{\text{inf}} \right) \]

Reactance 1 is in its high-frequency state

What resistance drives reactance 2?

R_{\text{inf}} is not added for clarity purposes
Assemble Time Constants for $b_2$

\[ b_2 = \frac{L_1}{R_{\text{inf}} + r_L} C_{\text{damp}} \left( R_{\text{damp}} + R_{\text{inf}} \right) \]
\[ + \frac{L_1}{R_{\text{inf}} + r_L} C_3 \left( r_C + R_{\text{inf}} \right) \]
\[ + C_{\text{damp}} \left( R_{\text{damp}} + r_L \parallel R_{\text{inf}} \right) C_3 \left( r_C + R_{\text{damp}} \parallel R_{\text{inf}} \parallel r_L \right) \]

\[ R_{\text{inf}} \to \infty \quad \rightarrow \quad b_2 = L_1 \left( C_{\text{damp}} + C_3 \right) + C_{\text{damp}} \left( R_{\text{damp}} + r_L \right) C_3 \left( r_C + r_L \parallel R_{\text{damp}} \right) \]

$\tau_{12} = \tau_1 \tau_2 \tau_3$

HF state
\[ \tau_3^{12} = C_3 \left( r_C + R_{\text{damp}} \parallel R_{\text{inf}} \right) \]

What resistance drives reactance 3?

\[ b_3 = \tau_1 \tau_2 \tau_3 \]

\[ b_2 = \tau_1 \tau_2^1 + \tau_1 \tau_3^1 + \tau_2 \tau_3^2 \]
Build the 3rd-Order Denominator

Gather time constants and rearrange to form $D(s)$

$$D(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3$$

$$b_1 = C_{damp} \left( R_{damp} + r_L \right) + C_3 \left( r_C + r_L \right)$$

$$b_2 = L_1 \left( C_{damp} + C_3 \right) + C_{damp} \left( R_{damp} + r_L \right) C_3 \left( r_C + r_L \parallel R_{damp} \right)$$

$$b_3 = \frac{L_1}{R_{inf}} C_{damp} \left( R_{damp} + R_{inf} \right) C_3 \left( r_C + R_{damp} \right) = L_1 C_{damp} C_3 \left( r_C + R_{damp} \right)$$

$$D(s) = 1 + s \left[ C_{damp} \left( R_{damp} + r_L \right) + C_3 \left( r_C + r_L \right) \right]$$

$$+ s^2 \left[ L_1 \left( C_{damp} + C_3 \right) + C_{damp} \left( R_{damp} + r_L \right) C_3 \left( r_C + r_L \parallel R_{damp} \right) \right]$$

$$+ s^3 \left[ L_1 C_{damp} C_3 \left( r_C + R_{damp} \right) \right]$$
Determine \( N(s) \) Swiftly with Inspection

\[ Z_1(s) = 0 \]
\[ Z_2(s) = 0 \]
\[ Z_3(s) = 0 \]

\[ L_1 \]
\[ C_3 \]
\[ R \_{damp} \]

\[ r_L \]
\[ r_C \]

Response is nulled
\[ V_T = 0 \]

Three zeros when
\[ Z_1(s_{z_1}) = 0 \quad Z_2(s_{z_2}) = 0 \quad Z_3(s_{z_3}) = 0 \]

\[ N(s) = \left( 1 + s \frac{L_1}{r_L} \right) \left( 1 + sr_C C_3 \right) \left( 1 + sR_{damp} C_{damp} \right) \]
Run a Sanity Check to Verify Results

\[ Z_{out}(s) = R_0 \frac{N(s)}{D(s)} \]

The complete low-entropy transfer function is thus

\[ Z_{out}(s) = R_0 \frac{1 + s r_C C_3 (1 + s R_{damp} C_{damp}) (1 + s \frac{L_1}{r_L})}{1 + s \left[ C_{damp} (R_{damp} + r_L) + C_3 (r_C + r_L) \right] + s^2 \left[ L_1 (C_{damp} + C_3) + C_{damp} (R_{damp} + r_L) C_3 (r_C + r_L \parallel R_{damp}) \right] + s^3 \left[ L_1 C_{damp} C_3 (r_C + R_{damp}) \right]} \]

The raw transfer function is

\[ Z_{out}(s) = Z_1(s) \parallel Z_2(s) \parallel Z_3(s) \]

Raw and full TF plots are superimposed: good to go!
Trying to Rewrite the Transfer Function

In this 3\textsuperscript{rd}-order system, it is difficult to find a canonical form such as

\[ Z_{\text{out}}(s) = R_0 \frac{N(s)}{\left(1 + \frac{s}{\omega_p}\right)\left(1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2\right)} \]

If \( r_L \) is larger than \( r_C \) and \( C_{\text{damp}}R_{\text{damp}} \) larger than \( C_3 r_L \)

\[ D(s) \approx 1 + sC_{\text{damp}}R_{\text{damp}} + s^2 L_1 \left(C_{\text{damp}} + C_3\right) + s^3 L_1 C_{\text{damp}} C_3 R_{\text{damp}} \]

Unfortunately, \( 1 + sC_{\text{damp}}R_{\text{damp}} \) does not dominate at low freq.

\rightarrow \text{Cannot factor the 3\textsuperscript{rd}-order denominator}
Neglect Parasitic Terms \( r_L \) and \( r_C \)

- Simplify the transfer function considering 0 \( r_L \) and \( r_C \)

\[
Z_{out}(s) = r_L + s\left[ C_{damp}(R_{damp} + r_L) + C_3(r_c + r_L) \right] + s^2\left[ L_1(C_{damp} + C_3) + C_{damp}(R_{damp} + r_L)C_3(r_c + r_L || R_{damp}) \right] + s^3\left[ L_4C_{damp}C_3(r_c + R_{damp}) \right]
\]

- Factor the \( L_1/r_L \) term

\[
Z_{out}(s) = r_L\frac{sL_1}{r_L + s\left[ C_{damp}(R_{damp} + r_L) + C_3r_L \right] + s^2\left[ L_1(C_{damp} + C_3) + C_{damp}(R_{damp} + r_L)C_3(r_L || R_{damp}) \right] + s^3\left[ L_4C_{damp}C_3R_{damp} \right]}
\]

- Have \( r_L \) go to zero

\[
Z_{out}(s) = sL_1\frac{1+sR_{damp}C_{damp}}{1+sC_{damp}R_{damp} + s^2L_1(C_{damp} + C_3) + s^3\left[ L_1C_{damp}C_3R_{damp} \right]}
\]

- Consider \( C_{damp} = nC_3 \)

\[
Z_{out}(s) = sL_1\frac{1+sR_{damp}nC_3}{1+sR_{damp}nC_3 + s^2L_1C_3(1+n) + s^3\left[ L_1nC_3^2R_{damp} \right]}
\]
Rearrange Expressions

- Determine the magnitude of this transfer function

\[
Z_{out}(j\omega) = \frac{L_1\omega^2 nR_{damp}C_3 - jL_1\omega}{(C_3L_1\omega^2 + C_3L_1\omega^2n - 1) - j(C_3R_{damp}\omega n)(1 - C_3L_1\omega^2)}
\]

\[
|Z_{out}(\omega)| = \frac{\sqrt{L_1\omega^2 nR_{damp}C_3}^2 + (L_1\omega)^2}{\sqrt{(C_3L_1\omega^2 + C_3L_1\omega^2n - 1)^2 + (C_3R_{damp}\omega n)^2}(1 - C_3L_1\omega^2)^2}
\]  \[\text{mag}_1\]

- Check with Middlebrook’s definitions

\[
R_0 = \sqrt{\frac{L_1}{C_3}} \quad Q = \frac{R_{damp}}{R_0} \quad p(s) = \frac{s}{\omega_0} \quad \omega_0 = \frac{1}{\sqrt{L_1C_3}}
\]

\[
Z_{out}(s) = \frac{p(s)[1+nQp(s)]}{1+nQp(s)+(1+n)p(s)^2+nQp(s)^3}
\]

\[
|Z_{out}(\omega)| = R_0 \frac{x\sqrt{1+n^2Q^2x^2}}{\sqrt{[1-(1+n)x^2]^2 + [xnQ(1-x^2)]^2}} \quad x = \frac{\omega}{\omega_0} \quad \text{mag}_2
\]
Always Run a Sanity Check

- Check if expressions are ok versus Mathcad® calculations

- Compare analytical results versus raw magnitude expression
Course Agenda

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The Damping Resistor Affects $Q$ and $\omega_0$

- The damping resistor value changes the resonant frequency

The circuit diagram shows a series RLC circuit with damping resistor $R_{damp}$, inductor $L_1$, and capacitor $C_3$.

The equations for $\omega_0$ are:

$$\omega_0 = \frac{1}{\sqrt{L_1C_3}}$$

$$\omega_0 = \frac{1}{\sqrt{L_1\left(C_3 + C_{damp}\right)}}$$

- There must be a $R_{damp}C_{damp}$ couple optimizing $Q$

- What is the minimum in the maximum (minmax) peaking of $|Z_{out}|$?
- For what optimum $Q$ value does it occur and at what frequency?
Changes Induced by Damping Resistor

- Plot magnitudes versus various $Q$ factors, $C_{damp} = 10C_3$

![Graph showing changes induced by damping resistor.](image)

- $R_{damp} = 0$
- $Q = 100$
- $R_{damp}$ decreases
- $Q_{opt}$
- $\omega_{opt}$
- $n = 10$

This point is independent of $Q$.

It is the point at which peaking is the lowest!
Optimum Peak Depends on $C_{damp}$

- Optimum $Z_{out}$ magnitudes versus different values of $n$

$$\left| Z_{out}(f, n) \right|$$

- 20 log \( \frac{V_{in}^2}{P_{out}} \)

- $Q = Q_{opt}$

- There is an optimum $R_{damp}$ and $n$ to match a given $Z_{out}$
Determine the Optimum Frequency

- The frequency at which the minmax occurs is immune to \( Q \)
  - Calculate the sensitivity of \( Z_{out} \) to \( Q \) and cancel it
  - Work on \( Z_{out}^2 \) to get rid of square roots

\[
\frac{d}{dQ}(Z_{out}(Q)^2) = \frac{d}{dQ}\left(R_0^2 \frac{x^2 (1 + n^2 Q^2 x^2)^2}{[1 - (1 + n) x^2]^2 + [xnQ(1 - x^2)]^2}\right)
\]

\[
\frac{d}{dQ}(Z_{out}(Q)^2) = \frac{2Qn^3x^6(nx^2 + 2x^2 - 2)}{D(Q)} = 0
\]

\[
nx^2 + 2x^2 - 2 = 0
\]

\[
x^2 (n + 2) = 2
\]

\[
x = \sqrt{\frac{2}{2 + n}}
\]

\[
x = \frac{\omega}{\omega_0}
\]

The point at which \( Q_{opt} \) occurs is:

\[
\omega_{opt} = \sqrt{\frac{2}{2 + n}} \omega_0 = \sqrt{\frac{2}{(2 + n)L_1 C_3}}
\]
Calculate the Magnitude at $\omega_{opt}$

- Update the magnitude definition to have $|Z_{out}|$ at $\omega_{opt}$

$$|Z_{out}(\omega)| = R_0 \frac{x \sqrt{1+n^2Q^2x^2}}{\sqrt{\left[1-(1+n)x^2\right]^2 + \left[xnQ(1-x^2)\right]^2}} \quad x = \frac{\omega_{opt}}{\omega_0} = \sqrt{\frac{2}{2+n}}$$

$$|Z_{out}(\omega_{opt})| = \frac{\sqrt{2(2+n)}}{n} R_0 = \frac{\sqrt{2(2+n)}}{n} \sqrt{\frac{L_1}{C_3}}$$

This is the value of $|Z_{out}|$ at $\omega_{opt}$

- We want to minimize $|Z_{out}|$ at $\omega_{opt}$
  - Differentiate $Z_{out}^2$ with respect to $x^2$, find the optimum $Q$
  - Replace $A = x^2$ and $\sqrt{A} = x$

$$\frac{d}{dA} \left[\frac{Z_{out}(A)}{R_0}\right]^2 = \frac{d}{dA} \frac{A(1+n^2Q^2A)}{\left[1-(1+n)A\right]^2 + \left[n\sqrt{A}Q(1-A)\right]^2} = 0$$
Determine the Optimum Value of $Q$

- Apply brute-force differentiation with Mathcad®

$$\frac{d}{dA} \left|_{R_0} \frac{Z_{out}(A)}{A^2} \right|^2 = \left( \frac{A Q_n^4 + 2A^3 Q^2 n^2 - A^2 Q^4 n^4 + 2A^2 Q^2 n^3 + A^2 n^2 + 2A^2 n + A^2 - 2A Q^2 n^2 - 1}{D} \right) = 0$$

Extract $Q$

$$\left( A Q_n \right)^4 + 2A^3 Q^2 n^2 - A^2 Q^4 n^4 + 2A^2 Q^2 n^3 + A^2 n^2 + 2A^2 n + A^2 - 2A Q^2 n^2 - 1 = 0$$

$$Q_{opt} = \sqrt{\frac{\sqrt{-A^3 n^5 \left( A^3 n - 2A - 2A^2 + 2A^3 - 2An + 2 \right) - An^2 + A^2 n^3 + A^3 n^2}}{A^2 n^4 - A^4 n^4}}$$

$$A = \frac{2}{2+n}$$

$$Q_{opt} = \sqrt{\frac{3n^2 + 10n + 8}{2n^2 (n+4)}} = \sqrt{\frac{(4+3n)(2+n)}{2n^2 (4+n)}}$$

Result from Dr Middlebrook
Apply the Technique to the Buck

- The target is to reduce the filter impedance peak to 0.7 \( \Omega \)

\[
R_0 = \frac{\sqrt{L_f}}{C_f}
\]

\[
\frac{|Z_{out}|_{mm}}{R_0} = \sqrt{\frac{2(2+n)}{n^2}}
\]

\[
Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} = 0.65
\]

\[
n = \frac{R_0 \left( R_0 + \sqrt{R_0^2 + 4\left(\left|Z_{out}\right|_{mm}\right)^2}\right)}{\left(\left|Z_{out}\right|_{mm}\right)^2} = 3.5
\]

\[
R_{damp} = R_0 Q_{opt} = 0.487 \Omega
\]

\[
C_{damp} = nC_f = 141 \ \mu F
\]

- This is a rather large capacitance value
  - An electrolytic capacitor and its ESR can do the job
  - Watch for temperature effects as ESR increases at low temp!
The overlap is gone for both impedances

Gain and phase distortion of $k$ are minimized
The output impedance should not be affected by the EMI filter.

\[
Z_{\text{th}(f)} \approx 20 \text{ dB}
\]

\[
20 \cdot \log \left( \frac{Z_{\text{th}\left(i \cdot 2\pi \cdot f_k\right)}}{1 \Omega} \right)
\]

\[
20 \cdot \log \left( \frac{Z_e\left(i \cdot 2\pi \cdot f_k\right)}{1 \Omega} \right)
\]

input impedance with 0-Ω R_{load}
Verify Results after Damping - Magnitude

- Check the resulting control-to-output transfer function

\[ 20 \cdot \log \left| T_{\text{filter}}(i \cdot 2\pi \cdot f_k) \right| \]

\[ 20 \cdot \log \left| T_{\text{filterM}}(i \cdot 2\pi \cdot f_k) \right| \]

- Magnitude distortion has disappeared after damping
Original phase margin is unaffected after damping

\[ \text{arg} \left( T_{\text{filter}} \left( i \cdot 2\pi \cdot f_k \right) \right) \cdot \frac{180}{\pi} \]

\[ \text{arg} \left( T_{\text{filterM}} \left( i \cdot 2\pi \cdot f_k \right) \right) \cdot \frac{180}{\pi} \]

\[ \angle T(f) \]
Closed-Loop Output Impedance

- Peaking effects of the EMI filter are now gone

\[ Z_{out,CL}(f) = 20 \log \left( \frac{Z_{outCLFD}(i \cdot 2\pi \cdot f_k)}{1\Omega} \right) \]

(dBΩ)

| \[ |Z_{out,CL}(f)| \] |
|---|
| 0 |
| -20 |
| -40 |
| -60 |
| -80 |
| -100 |

\[ 10 \quad 100 \quad 1 \times 10^3 \quad 1 \times 10^4 \quad 1 \times 10^5 \]

\( f_k \)

Damped filter

Undamped filter

Public Information
Christophe Basso – Input Filter Interactions

ON Semiconductor®
Opting for a Different Strategy

- What if you only try to get rid of the overlap, ignoring $Z_D$ and $Z_N$?
  - Plot the closed-loop input impedance and build margin (10 dB)

\[
20 \log \left( \frac{Z_{th}(i \cdot 2\pi f_k)}{1 \Omega} \right)
\]

\[
20 \log \left( \frac{Z_{in,CL}(i \cdot 2\pi f_k)}{1 \Omega} \right)
\]

\[
Z_{th}(f)
\]

\[
Z_{in,CL}(f)
\]

\[
10 \text{ dBΩ}
\]

\[
3.3 \Omega
\]
Calculate the New Damping Elements

Different damping elements are now required

\[
R_n = \frac{R_0 + \sqrt{R_0^2 + 4(|Z_{out}|_{mm})^2}}{(|Z_{out}|_{mm})^2} = 0.5
\]

\[
Q_{opt} = \sqrt{\frac{(4 + 3n)(2 + n)}{2n^2(4 + n)}} = 2.4
\]

\[
R_{damp} = R_0 Q_{opt} = 1.8 \Omega
\]

\[
C_{damp} = nC_f = 20 \mu F
\]

Filter output impedance

Plot includes all ohmic losses
The new filter effect can be observed when it resonates.

\[ 20 \log \left| T_{\text{filter}}(i \cdot 2\pi \cdot f_k) \right| \]

\[ 20 \log \left| T_{\text{filterM}}(i \cdot 2\pi \cdot f_k) \right| \]

Gain distortion is noticeable before crossover.
Phase distorsion appears but do not jeopardize phase margin

Original phase margin is unaffected in this case
Peaking can be observed but it remains limited.
Both damping strategies can be tested with SPICE

\[ R_{\text{damp}} = 0.487 \, \Omega \quad \text{or} \quad R_{\text{damp}} = 1.8 \, \Omega \]
\[ C_{\text{damp}} = 141 \, \mu F \quad \text{or} \quad C_{\text{damp}} = 20 \, \mu F \]

The load is stepped from 50 to 100% in 1 \( \mu s \)
Oscillatory but stable response to a load step

\[ R_{damp} = 1.8 \, \Omega \]
\[ C_{damp} = 20 \, \mu F \]

\[ R_{damp} = 0.487 \, \Omega \]
\[ C_{damp} = 141 \, \mu F \]

\[ v_{out}(t) \]
This is a simple model compensated by the type III structure.

You can verify the step response but also the filtered signature.
Oscillations in the Switched Model

- Oscillations are present but not too disturbing

\[ R_{damp} = 1.8 \Omega \]
\[ C_{damp} = 20 \mu F \]

\[ v_{out}(t) \]

\[ R_{damp} = 0.487 \Omega \]
\[ C_{damp} = 141 \mu F \]

\[ v_{out}(t) \]
Input Current Signature is Good

- No difference in the signature – amplitude is within specs

\[ I_{in, peak} = 14.8 \text{ mA} \]

\[ R_{damp} = 0.487 \ \Omega \]

\[ C_{damp} = 141 \ \mu F \]

\[ R_{damp} = 1.8 \ \Omega \]

\[ C_{damp} = 20 \ \mu F \]
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This LED driver featuring PFC requires the insertion of a filter.
Plot the Impedance Ratio $Z_{out}/Z_{in}$

Analysis reveals a negative phase margin: stability issue

$$\angle \frac{Z_{th}(f)}{Z_{in}(f)} = -233^\circ$$

$$|\frac{Z_{th}(f)}{Z_{in}(f)}| = 1$$
A Simulation Shows Oscillations

- The negative phase margins brings a diverging filter voltage

\[ v_{in}(t) \]

- \( v_{in}(t) \) vs. time
- Voltage levels: 120 V, 90 V
- Capacitance: \( C_1 = 0 \) pF
- Time intervals: 28.9m, 86.7m, 144m, 202m, 260m
Act on the PFC Input Voltage

- Reduce the phase stress by inserting a zero in the PFC chain

\[ H_0 = \frac{R_2}{R_1 + R_2} \]

\[ \tau_1 = C_1 R_1 \quad \tau_2 = \left( R_1 \parallel R_2 \right) \left( C_1 + C_2 \right) \]

\[ H(s) = H_0 \frac{1 + s \tau_1}{1 + s \tau_2} = H_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \]

\[ \omega_z = \frac{1}{\tau_1} \quad \omega_p = \frac{1}{\tau_2} \]
Adding a zero in the PFC control input brings phase margin

Phase margin is 45°
Adding a zero builds phase margin and tames oscillations.
Practical Experiments

- Placing a zero at 2.5 kHz (22-pF cap.) stops oscillations.
- However, PFC operations are affected: need to damp the filter!
Explore Another Option and Damp Filter

\[ R_0 = \sqrt{\frac{L_1}{C_3}} \]
\[ \left| Z_{out} \right|_{mm} = \frac{\sqrt{2(2+n)}}{n^2} \]
\[ n = \frac{R_0 \left( R_0 + \sqrt{R_0^2 + 4 \left| Z_{out} \right|_{mm}^2} \right)}{\left| Z_{out} \right|_{mm}^2} \]

\[ Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} \]
\[ R_{damp} = R_0 Q_{opt} \]

\[ |Z_{out}|_{mm} \quad \text{This is the minimum of } Z_{out} \text{ peaking to avoid overlap} \]

\[ |Z_{out}|_{mm} = k \cdot \frac{V_{in}^2}{P_{out}} \]
\[ \text{For a 40-W output and a 108-V input:} \]
\[ |Z_{out}|_{mm} = 291 \Omega \]
\[ \text{Assume 10\% margin, } k = 0.9 \quad |Z_{out}|_{mm} = 262 \Omega \]
\[ n = 0.584 \]
\[ Q_{opt} = 2.18 \]
\[ C_{damp} = n \cdot 220 \text{ nF} = 0.128 \mu F \]
\[ R_{damp} = R_0 Q_{opt} = 147 \Omega \]
Plots with $L_1 = 1 \text{ mH}$ and $C_3 = 220 \text{ nF}$

- Check absence of overlap with Mathcad®

Minimum resistance at $V_{in} = 108 \text{ V rms}$

No overlap

$R_{damp} = 146 \Omega$ and $C_{damp} = 0.128 \mu\text{F}$
Check Results on the Simulation Template

- Simulation results $V_{in} = 108$ V rms $P_{out} = 40$ W: no overlap

$$R_{damp} = 180 \ \Omega \text{ and } C_{damp} = 0.1 \ \mu F$$

Graph showing $\angle \frac{Z_{th}(f)}{Z_{in}(f)}$ and $\left| \frac{Z_{th}(f)}{Z_{in}(f)} \right|$ with a 5-dB margin.
Final $Q$ is affected by several other factors:

- PCB traces, dielectric losses in the caps but also iron losses in the inductor.
- True peaking is often lower and gives design margin.

No damping
7.5 mH and 220 nF (no $C_2$) after bridge. $P_{out} > 40$ W
$V_{in} = 120$ V rms

Damping 100 nF + 180 $\Omega$
7.5 mH and 220 nF (no $C_2$) after bridge. $P_{out} > 40$ W
$V_{in} = 120$ V rms

Curves and experiments by J. Turchi
Course Agenda

- A Switching Regulator as a Load
- EMI Filter Impact
- An Introduction to FACTs
- Buck Converter Input/Output Impedances
- Filtering the Input Current
- Damping the Filter
- Optimum Component Selection
- A Practical Case Study
- Cascading Converters
Cascading Converters

- When cascading converters, impedances matter

![Cascading Converters Diagram]

- The system can be also be modeled by a gain $T_M$

$$V_{in}(s) = V_{th}(s) \frac{1}{1 + \frac{Z_{th}(s)}{Z_{in}(s)}}$$
Use SPICE Models First

Simulate the boost converter output impedance

Closed-loop $Z_{out}$

11-15 V/24 V – 3 A
Use SPICE Models First

- Simulate the buck converter input impedance

![Circuit Diagram]

Closed-loop $Z_{in}$

$|Z_{in}(f)|$

(dBΩ)

10 100 1k 10k 100k 1Meg

24 V/5 V – 12 A

24 V/5 V – 12 A

Public Information
Christophe Basso – Input Filter Interactions

ON Semiconductor®
Compare Magnitude Curves

\[ |Z_{in}(f)| \leq -1.0 \text{ (dBΩ)} \]

\[ \frac{|Z_{out}(f)|}{|Z_{in}(f)|} < 1 \]

No overlap

\[ |Z_{out}(f)| \]

(dBΩ)
Compare Phase Curves

\[ \angle Z_{\text{in}}(f) \]

\[ \angle Z_{\text{out}}(f) \]

Neg. up to 1 kHz

Public Information
Christophe Basso – Input Filter Interactions
There is no gain, \[ |Z_{out}| \ll |Z_{in}| \]. The system is stable.
Cascade Converters for Simulation

Boost
- L1 33u 11.0V
- R5 7m
- X3 PWMCM
  L = 33u
  Fs = 200k
  Ri = 300m
  Se = 82k
- Vin 11
- dc_boost
- VterBoost
- C2 282p
- R2 50k
- R1 1.16k
- V2 2.37V
- X2 AMPSIMP
  VHIGH = 10
  VLOW = 10m
- Vout 2.37V

Buck
- VoBoost
- C3 820u
- L2 8u
- C5 820u
- R10 18m
- R4 4m
- Resr 14m
- Cout 820u
- Iout
- VterBuck
- C6 1n
- R6 1k
- C1 3.5n
- R7 1m
- V3 2.5

Public Information
Christophe Basso – Input Filter Interactions
Transient response is good for the buck and boost is stable.

\[ v_{out}(t) \]

- **Buck**
  - \( I_{out} \) from 5 to 12 A in 1 µs

- **Boost**
  - \( v_{out}(t) \)

\[ 23.90 \leq v_{out}(t) \leq 24.10 \]

\[ 5.00 \leq v_{out}(t) \leq 5.10 \]
Input impedance measurement requires a dedicated circuit

\[ Z_{in}(s) = -R_{\text{sense}} \frac{V_{\text{in}}(s)}{V_{\text{sense}}(s)} \]

You need to inject enough current to observe a modulated $V_{\text{out}}$

$Z_{\text{out}}(s) = -R_{\text{sense}} \frac{V_{\text{out}}(s)}{V_{\text{sense}}(s)}$

“Designing Control Loops for Linear and Switching Power Supplies”, C. Basso, Artech House, 2012
“Input Filter Considerations in Design and Applications of Switching Regulators”, R. D. Middlebrook, IAS 1976
Conclusion

- Incremental input resistance of switching converter is negative
- Inserting a $LC$ filter impacts the switching converter performance
- You have two design strategies
  - you design the filter together with the converter
  - you add a filter to an unknown-content converter
- You must determine open- and closed-loop input impedance
- Cascading converters requires knowledge of $Z_{out}$ and $Z_{in}$
- Test in the lab. dynamic responses when the filter is added
- Explore how damping is maintained at temperature extremes

Merci !
Thank you!
Xiè-xie!