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Modeling the Control Voltage to Duty Ratio Block

Using a Voltage Mode Stage to Model a CCM Current Mode Buck

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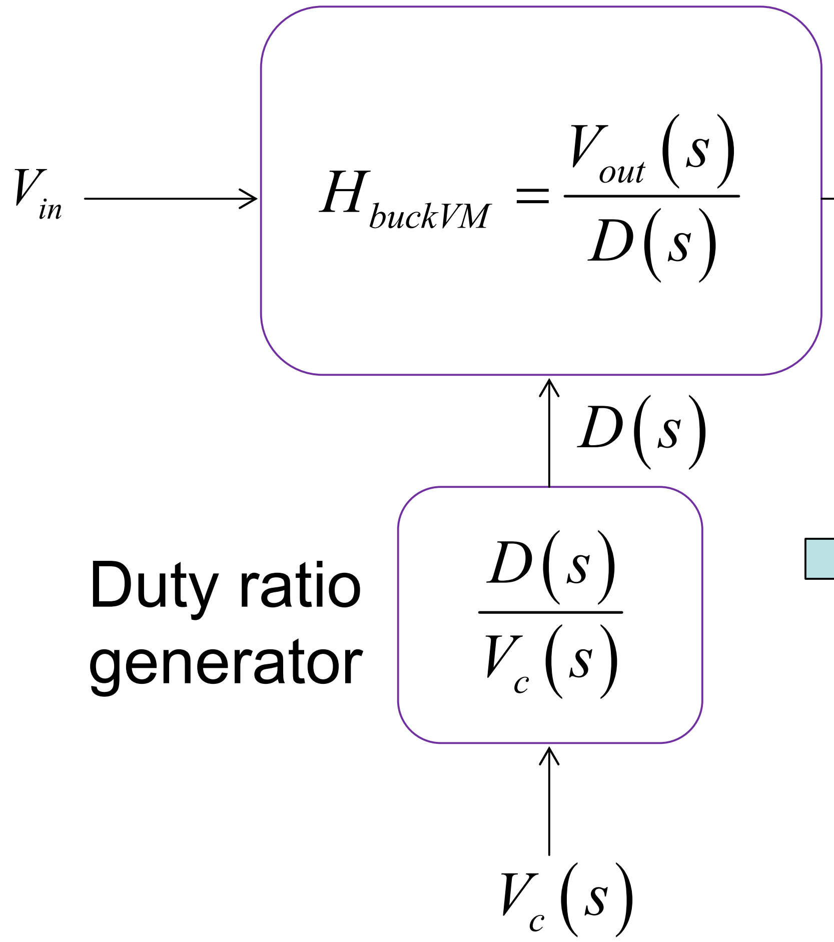
Modeling current mode converters with an added generator:
Keep the VM transfer function $V_{out}(f)/D(f)$
→ Model only the extra generator $D(f)/V_c(f)$



$$H_{buckCCM} = \boxed{\frac{V_{out}(s)}{D(s)}}\boxed{\frac{D(s)}{V_c(s)}}$$

Known VM TF

Duty ratio gen. TF

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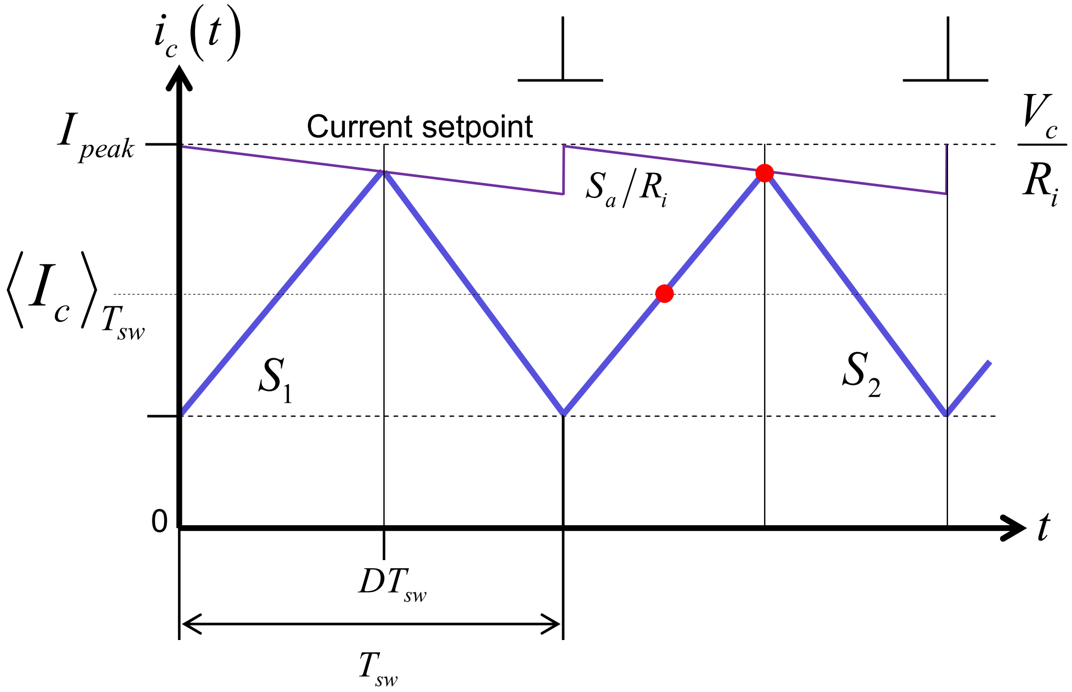
Terminal c current in the PWM switch model, CCM operation
We can extract the duty ratio definition from the average current in terminal c



$$I_c = \frac{V_c}{R_i} - \frac{S_a}{R_i} DT_{sw} - \frac{V_{ac}}{2L} DT_{sw} \quad \Rightarrow \quad D = \frac{F_{sw}\left(V_c - R_i I_c\right)}{S_a + \dfrac{R_i V_{ac}}{2L}}$$

Then extract the small-signal coefficients for all variables

$$\hat{d} = \frac{\partial\left(\dfrac{F_{sw}\left(V_c - R_i I_c\right)}{S_a + \dfrac{R_i V_{ac}}{2L}}\right)}{\partial V_c}\hat{v}_c + \frac{\partial\left(\dfrac{F_{sw}\left(V_c - R_i I_c\right)}{S_a + \dfrac{R_i V_{ac}}{2L}}\right)}{\partial I_c}\hat{i}_c + \frac{\partial\left(\dfrac{F_{sw}\left(V_c - R_i I_c\right)}{S_a + \dfrac{R_i V_{ac}}{2L}}\right)}{\partial V_{ac}}\hat{v}_{ac}$$

$$\hat{d} = k_c \hat{v}_c + k_{ic}\hat{i}_c + k_{ac}\hat{v}_{ac}$$

Must express these variables

$$k_c = \frac{F_{sw}}{S_a + \dfrac{R_i V_{ac}}{2L}} \qquad k_{ic} = -\frac{F_{sw} R_i}{S_a + \dfrac{R_i V_{ac}}{2L}} \qquad k_{ac} = -\frac{F_{sw} R_i \left(V_c - I_c R_i\right)}{2L\left(S_a + \dfrac{R_i V_{ac}}{2L}\right)^2}$$

For a buck $I_c = \dfrac{V_{out}}{R_L}$ $\quad V_{ac} = V_{in} - V_{out}$

$V_{in} := 15V \quad V_{out} := 5.03V \quad C_{out} := 100\mu F \quad r_C := 0.1\Omega \quad L_1 := 100\mu H \quad R_L := 1\Omega$

$V_{ac} := V_{in} - V_{out} = 9.97V \qquad I_c := \frac{V_{out}}{R_L} = 5.03A \qquad D := \frac{V_{out}}{V_{in}} = 0.335$

$V_c := 1.3V \quad F_{sw} := 100kHz \quad S_a := 0 \quad R_i := 0.25\Omega$

$$k_c := \frac{F_{sw}}{S_a + \frac{R_i \cdot V_{ac}}{2 \cdot L_1}} = 8.024\frac{1}{V} \qquad k_{ic} := \frac{F_{sw} \cdot R_i}{S_a + \frac{R_i \cdot V_{ac}}{2 \cdot L_1}} = 2.006\frac{1}{A} \qquad k_{ac} := \frac{F_{sw} \cdot R_i \cdot (V_c - I_c \cdot R_i)}{2 \cdot L_1 \cdot \left(S_a + \frac{R_i \cdot V_{ac}}{2 \cdot L_1}\right)^2} = 0.034\frac{1}{V}$$

$$I_c(s) = \frac{V_{(c)}(s) - I_c(s) Z_{eq}(s)}{sL}$$

For a buck

$$V_{(c)}(s) = V_{in} D(s) + \cancel{V_{in}(s) D_0}$$

$$I_c(s) = \frac{V_{in} D(s) - I_c(s) Z_{eq}(s)}{sL} \longrightarrow I_c(s)\left(sL + Z_{eq}(s)\right) = V_{in} D(s)$$

$$I_c(s) = \frac{V_{in} D(s)}{sL + Z_{eq}(s)}$$

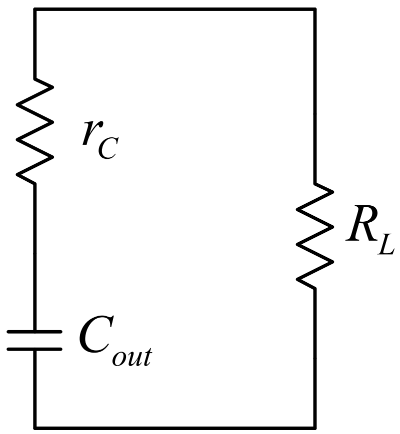
$$V_{ac}(s) = \cancel{V_{(a)}(s)} - V_{(c)}(s) = -V_{in} D(s)$$

$$D(s)=k_c V_c(s)-k_{ic}\frac{V_{in}D(s)}{sL+Z_{eq}(s)}+k_{ac}D(s)V_{in}$$

$$D(s)\left[1+k_{ic}\frac{V_{in}}{sL+Z_{eq}(s)}-k_{ac}V_{in}\right]=k_c V_c(s)$$

$$H(s)=\frac{D(s)}{V_c(s)}=\frac{k_c}{1+k_{ic}\dfrac{V_{in}}{sL+Z_{eq}(s)}-k_{ac}V_{in}}$$

$Z_{eq}(s)=?$



$$Z_{eq}(s)=R_L\frac{1+sr_C C_{out}}{1+sC_{out}(r_C+R_L)}$$

Develop $H(s)$, replace and rearrange…

Same location as LC filter poles

$$H(s)=\frac{R_L k_c}{\left(R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}\right)}\frac{\boxed{1+s\left(\frac{L+C_{out}R_L r_c}{R_L}\right)+s^2 LC_{out}\left(\frac{R_L+r_C}{R_L}\right)}}{1+s\left(\frac{L+C_{out}R_L r_C-LV_{in}k_{ac}+C_{out}R_L V_{in}k_{ic}+C_{out}V_{in}k_{ic}r_C-C_{out}R_L V_{in}k_{ac}r_C}{R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}}\right)+s^2\left(\frac{C_{out}LR_L+C_{out}Lr_C-C_{out}LR_L V_{in}k_{ac}-C_{out}LV_{in}k_{ac}r_C}{R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}}\right)}$$

$$H(s)=H_0\frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2}\qquad a_1=\frac{L+C_{out}R_L r_c}{R_L}\qquad a_2=LC_{out}\left(\frac{R_L+r_C}{R_L}\right)$$

$$b_1=\frac{L+C_{out}R_L r_C-LV_{in}k_{ac}+C_{out}R_L V_{in}k_{ic}+C_{out}V_{in}k_{ic}r_C-C_{out}R_L V_{in}k_{ac}r_C}{R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}}$$

$$b_2=\frac{C_{out}LR_L+C_{out}Lr_C-C_{out}LR_L V_{in}k_{ac}-C_{out}LV_{in}k_{ac}r_C}{R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}}$$

$$H_0=\frac{R_L k_c}{R_L+V_{in}k_{ic}-R_L V_{in}k_{ac}}\qquad Q_b=\frac{1}{b_1\omega_0}=\frac{\sqrt{b_2}}{b_1}\qquad \omega_0=\frac{1}{\sqrt{a_2}}$$

$$Q := \frac{\sqrt{\frac{C_{out}\cdot L_1\cdot R_L + C_{out}\cdot L_1\cdot r_C - C_{out}\cdot L_1\cdot R_L\cdot V_{in}\cdot k_{ac} - C_{out}\cdot L_1\cdot V_{in}\cdot k_{ac}\cdot r_C}{R_L + V_{in}\cdot k_{ic} - R_L\cdot V_{in}\cdot k_{ac}}}}{\frac{L_1 + C_{out}\cdot R_L\cdot r_C - L_1\cdot V_{in}\cdot k_{ac} + C_{out}\cdot R_L\cdot V_{in}\cdot k_{ic} + C_{out}\cdot V_{in}\cdot k_{ic}\cdot r_C - C_{out}\cdot R_L\cdot V_{in}\cdot k_{ac}\cdot r_C}{R_L + V_{in}\cdot k_{ic} - R_L\cdot V_{in}\cdot k_{ac}}} = 0.12$$

Low-Q approximation holds

$$\omega_0 := \frac{1}{\sqrt{\frac{C_{out}\cdot L_1\cdot R_L + C_{out}\cdot L_1\cdot r_C - C_{out}\cdot L_1\cdot R_L\cdot V_{in}\cdot k_{ac} - C_{out}\cdot L_1\cdot V_{in}\cdot k_{ac}\cdot r_C}{R_L + V_{in}\cdot k_{ic} - R_L\cdot V_{in}\cdot k_{ac}}}} = 7.556\times 10^4 \frac{1}{s}$$

$$\Rightarrow \quad D(s) = 1 + b_1 s + b_2 s^2 = 1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2 \approx \left(1 + \frac{s}{\omega_{p_1}}\right)\left(1 + \frac{s}{\omega_{p_2}}\right)$$

$$f_{p_1} = \frac{\omega_0 Q}{2\pi} = 1.44\text{ kHz} \qquad f_{p_2} = \frac{\omega_0}{Q 2\pi} = 100\text{ kHz}$$

First pole

Second pole

The final plant transfer function of the CCM-operated buck is:

$$H_{buck}(s) = \boxed{\frac{V_{out}(s)}{D(s)}} \boxed{\frac{D(s)}{V_c(s)}}$$

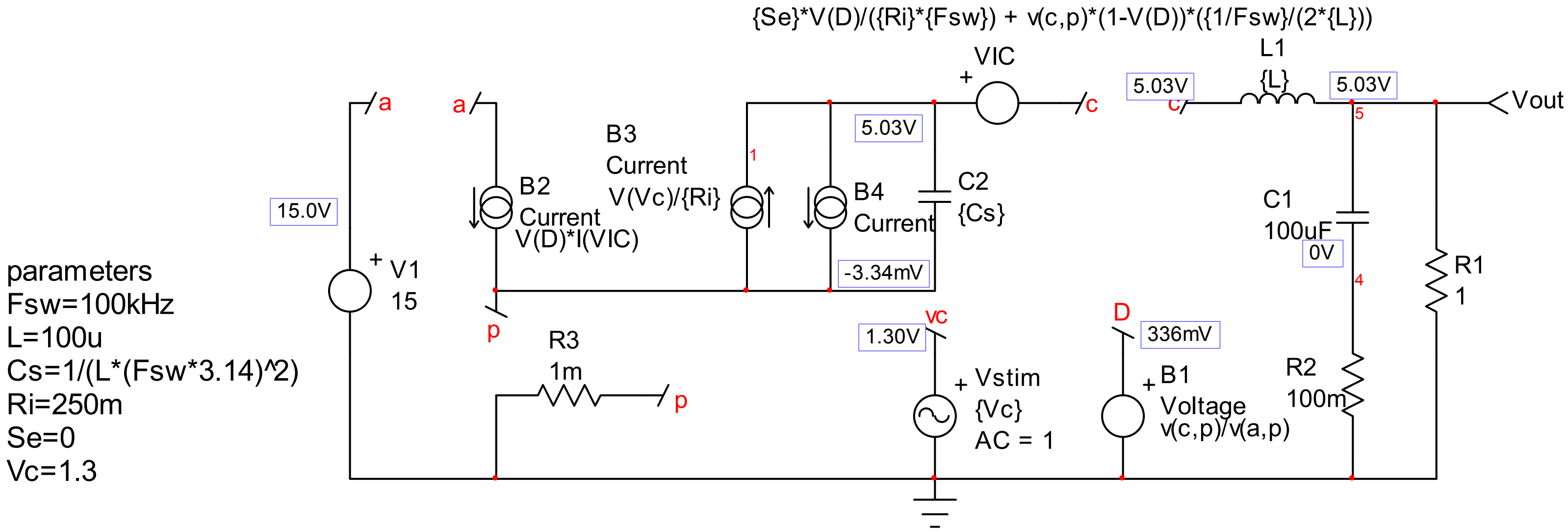
Voltage mode Buck TF

Duty ratio Factory TF

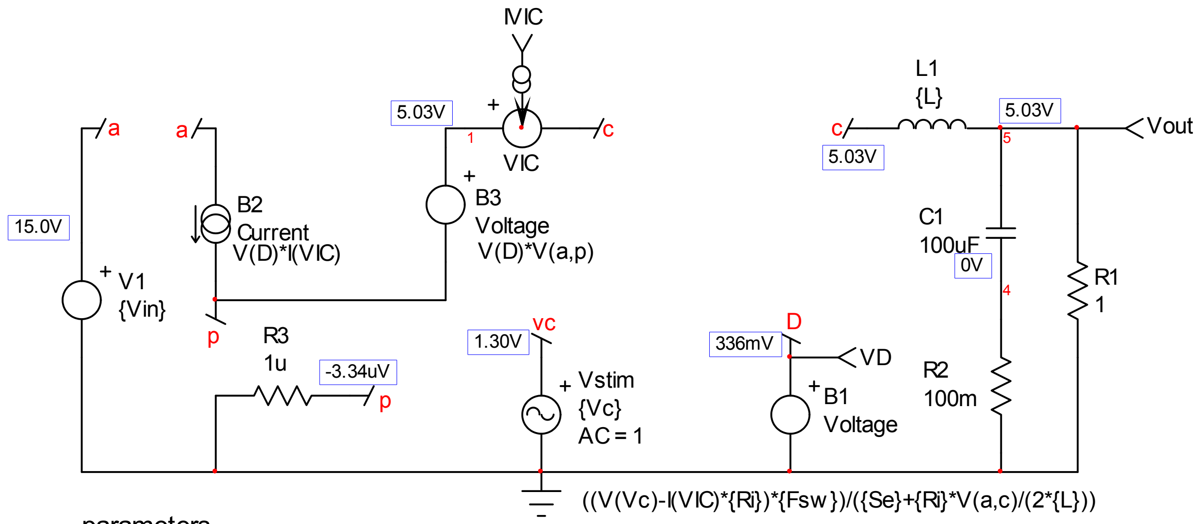
$$H_{buck}(s) = V_{in} \frac{1 + s r_C C_{out}}{\cancel{1 + \frac{s}{\omega_1 Q_1} + \left(\frac{s}{\omega_1}\right)^2}} H_0 \frac{\cancel{1 + \frac{s}{\omega_1 Q_1} + \left(\frac{s}{\omega_1}\right)^2}}{\left(1 + \frac{s}{\omega_{p_1}}\right)\left(1 + \frac{s}{\omega_{p_2}}\right)}$$

$$\Rightarrow \quad H_{buck}(s) \approx V_{in} H_0 \frac{1 + s r_C C_{out}}{\left(1 + \frac{s}{\omega_{p_1}}\right)\left(1 + \frac{s}{\omega_{p_2}}\right)}$$

Original large-signal CCM CM PWM switch model in a CCM buck converter



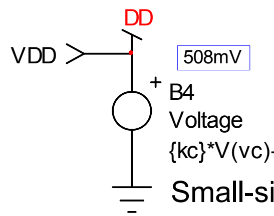
Simulation test model with duty ratio factory CCM buck converter



Large-signal duty ratio generator

parameters

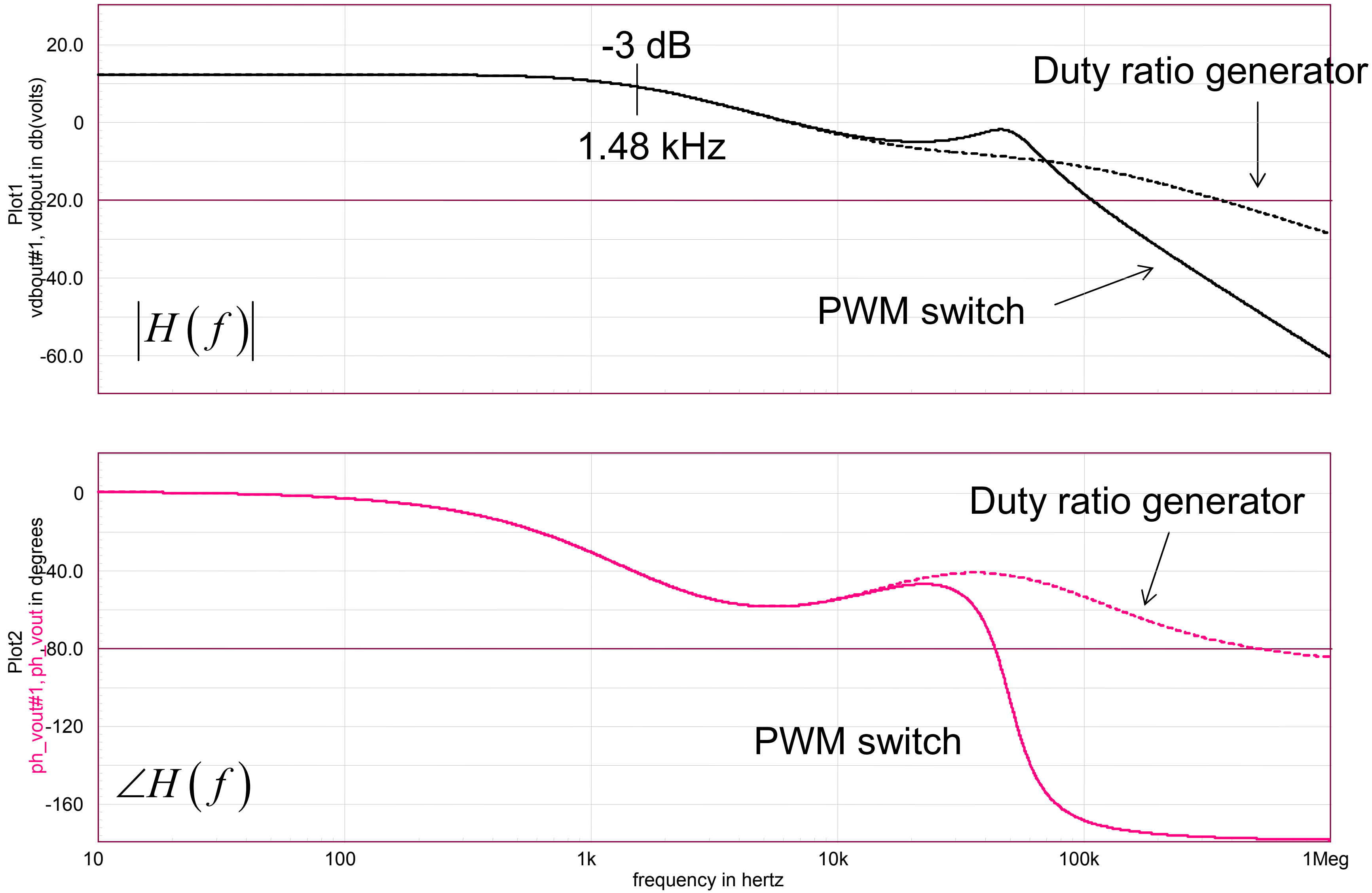
Vin=15
Vout=5.03
Fsw=100kHz
L=100u
Ri=250m
Se=0
Vc=1.3
RL=1
Vac=Vin-Vout
Ic=Vout/RL



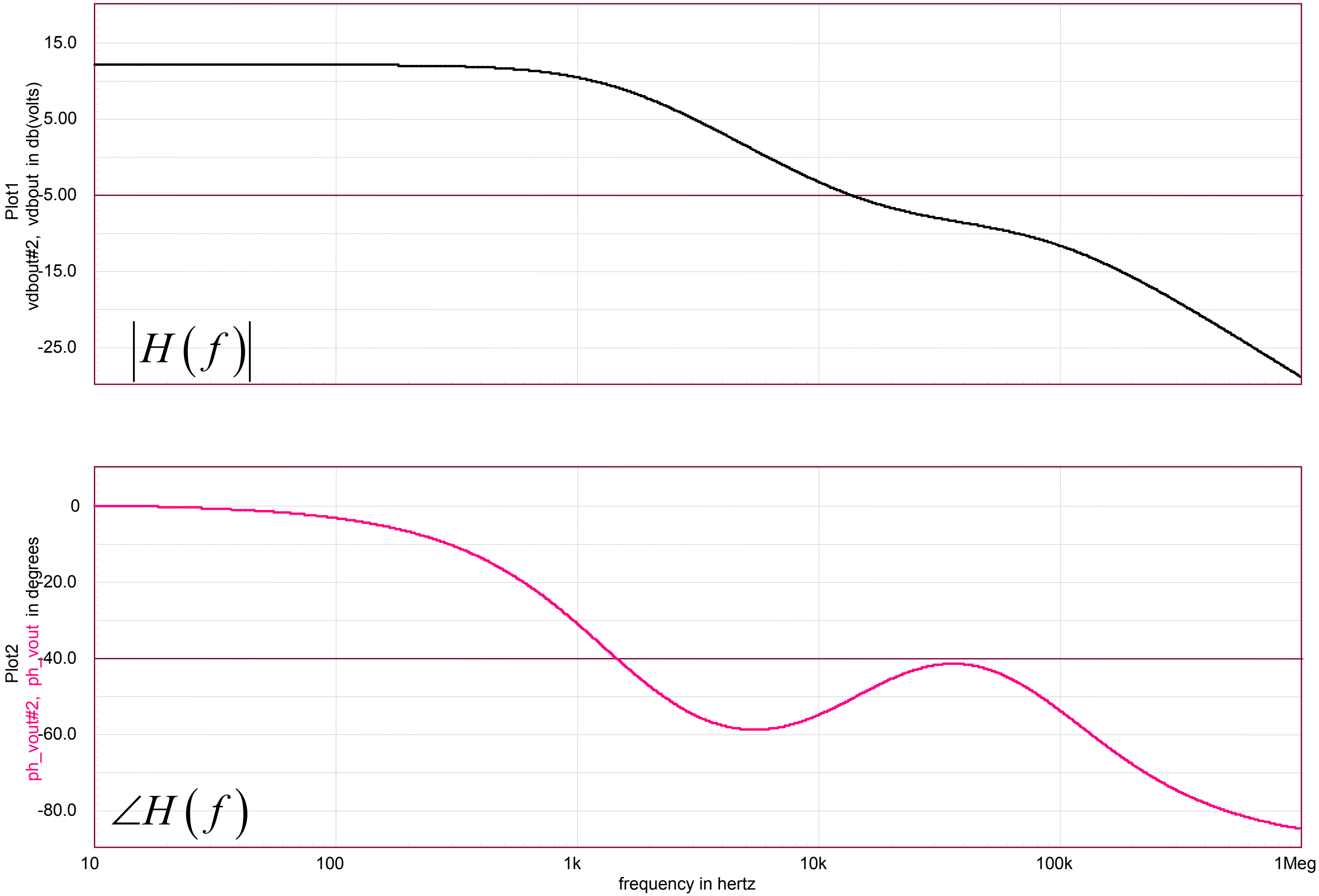
Small-signal duty ratio generator

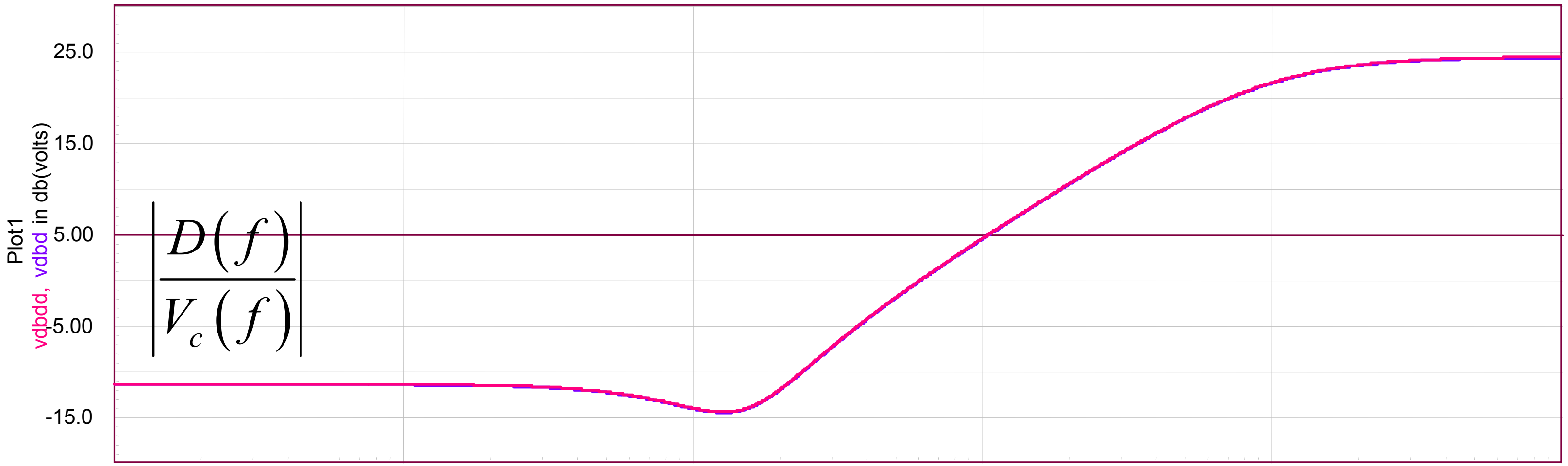
kc=Fsw/(Se+Ri*Vac/(2*L))
kic=(Fsw*Ri)/(Se+Ri*Vac/(2*L))
kacc=Fsw*Ri*(Vc-Ic*Ri)
kac=(Fsw*Ri)*(Vc-Ic*Ri)/(2*L*(Se+Ri*Vac/(2*L))^2)

Power stage response alone – hi-freq. response differs because of sub-harmonic poles

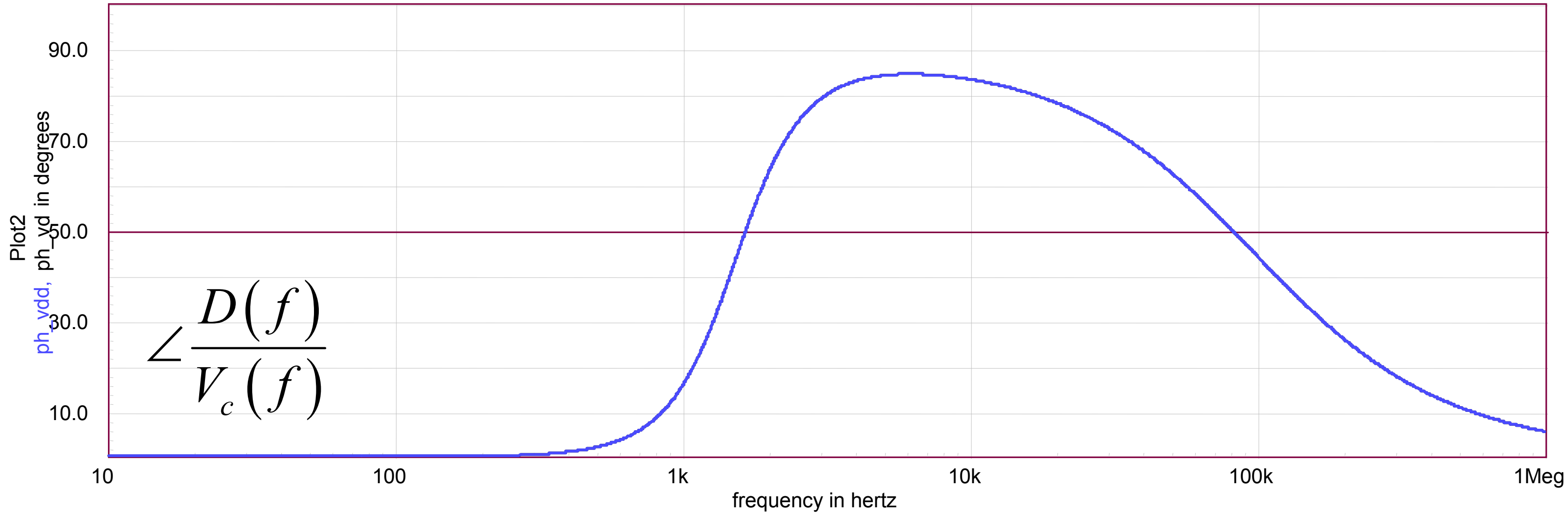


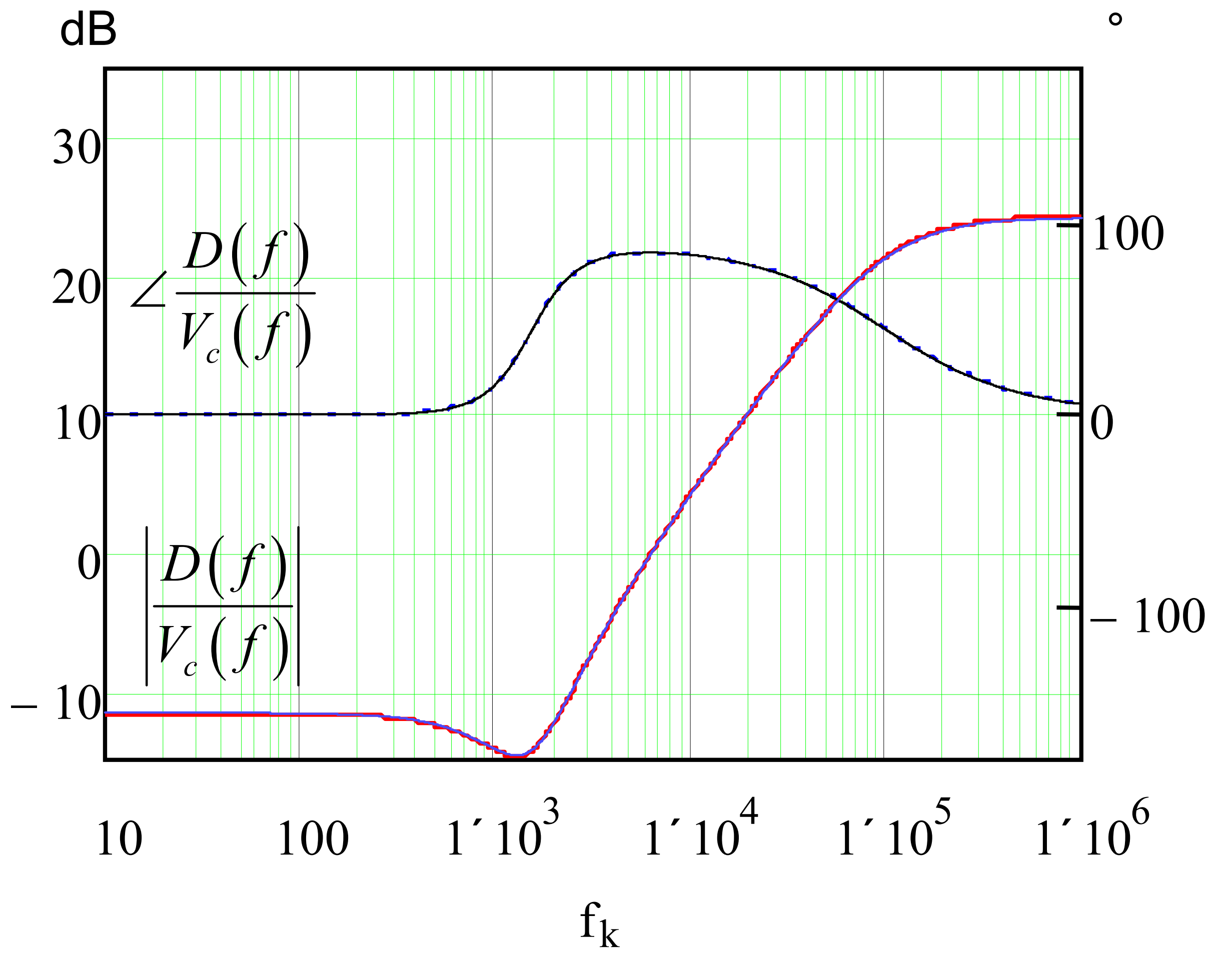
PWM switch without CS cap. versus duty ratio generator: no difference



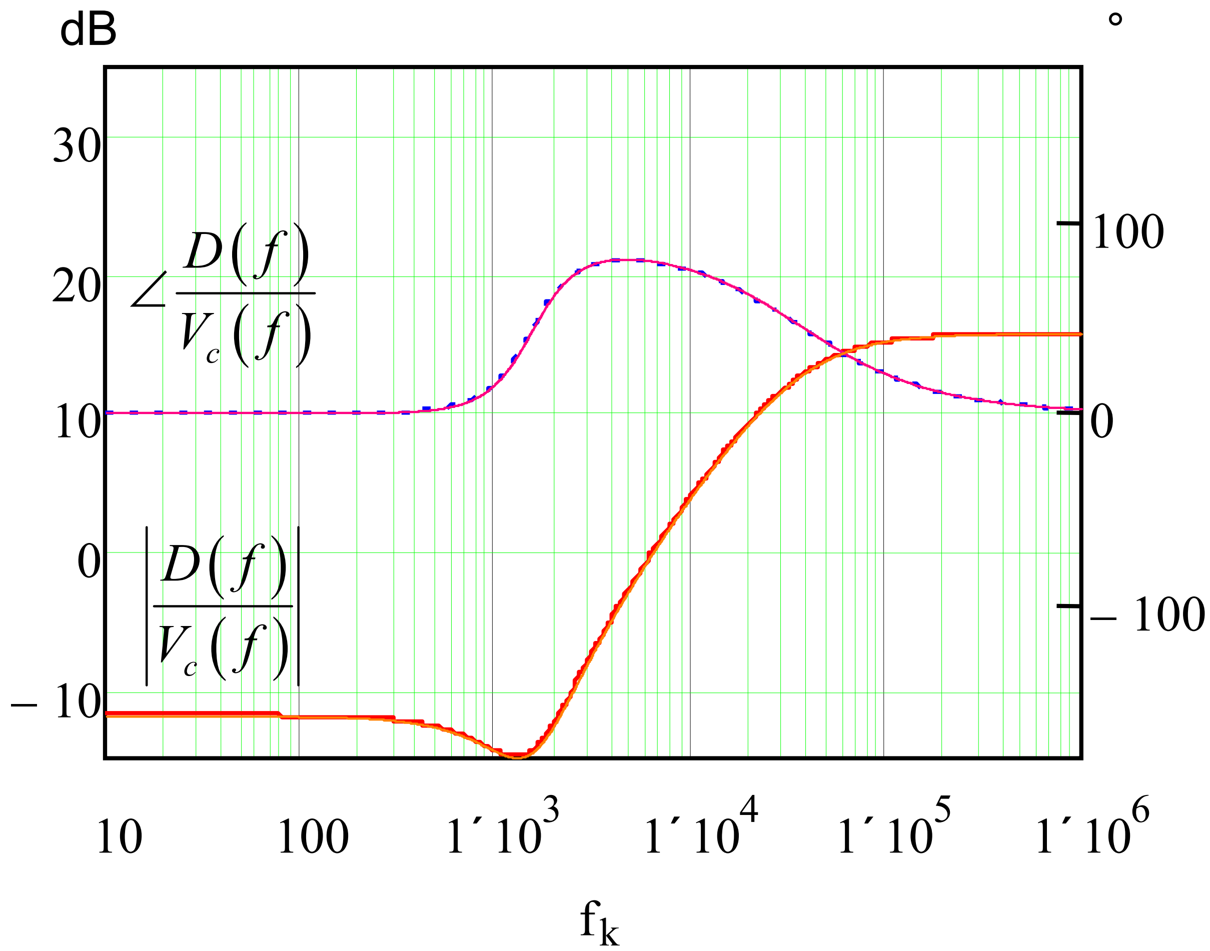


Large and linearized response of the V_c to D generator



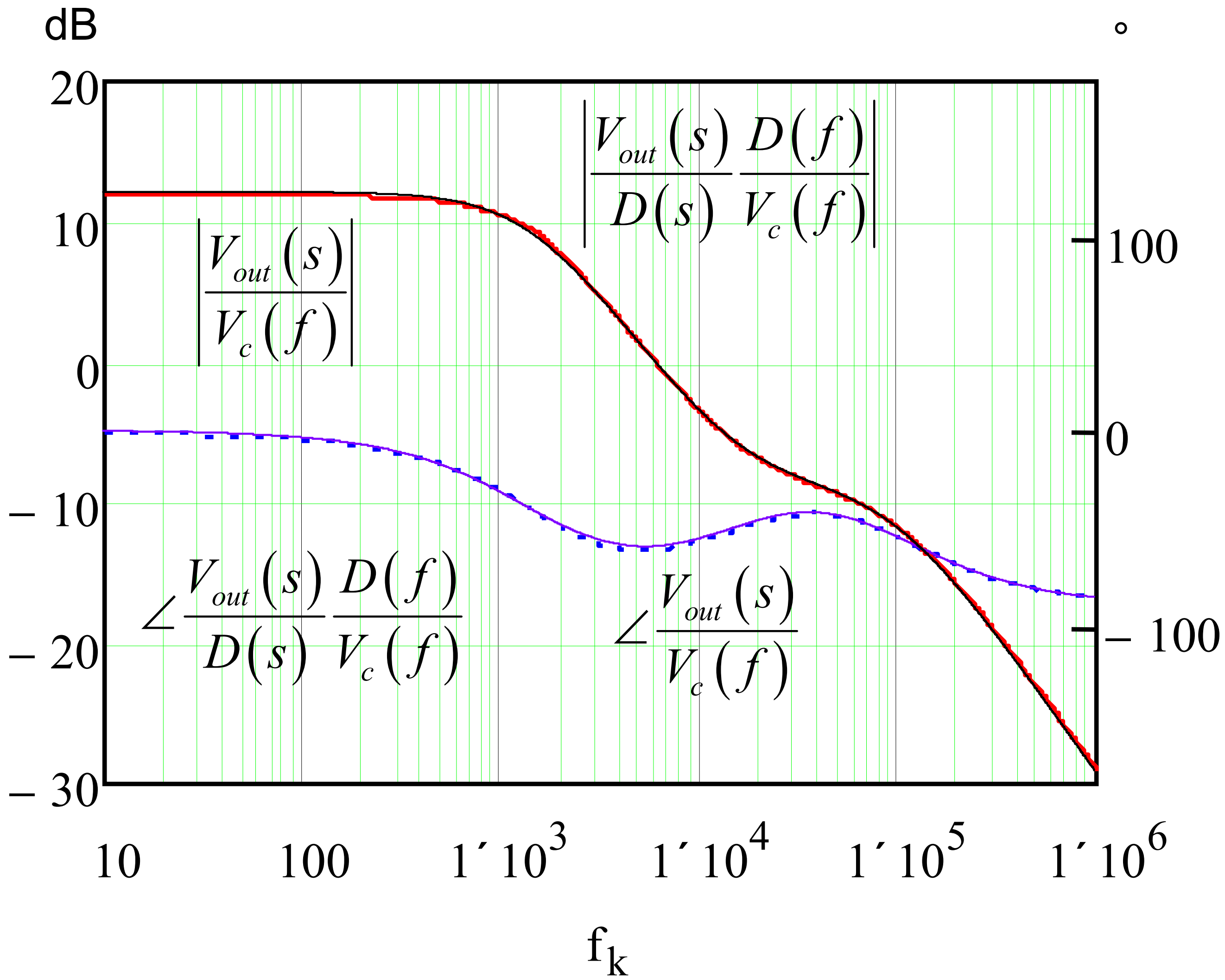


Ac response of duty ratio generator, SPICE versus Mathcad, $S_a = 0$



Ac response of duty ratio generator, SPICE versus Mathcad, S_a = 10 kV/s

Power stage CCM Buck CM final response of cascaded blocks versus large-signal model – S_a = 0



Power stage CCM Buck CM final response of cascaded blocks versus large-signal model – S_a = 10 kV/s