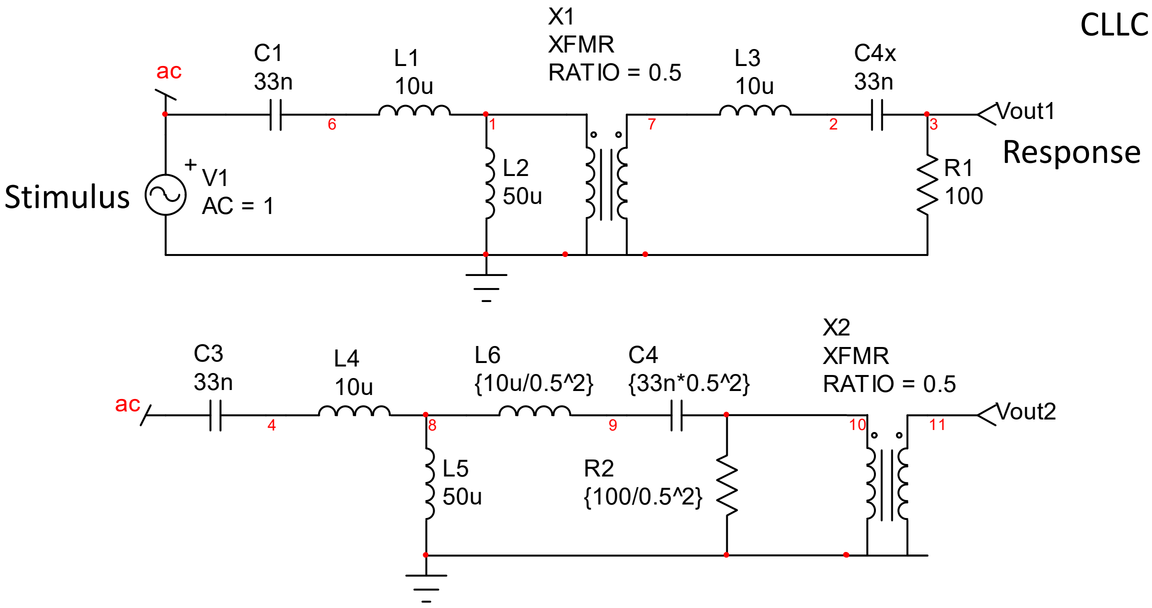
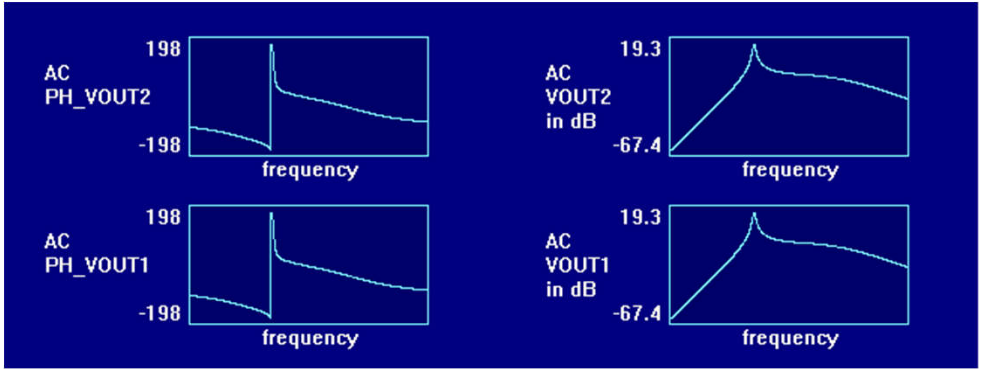
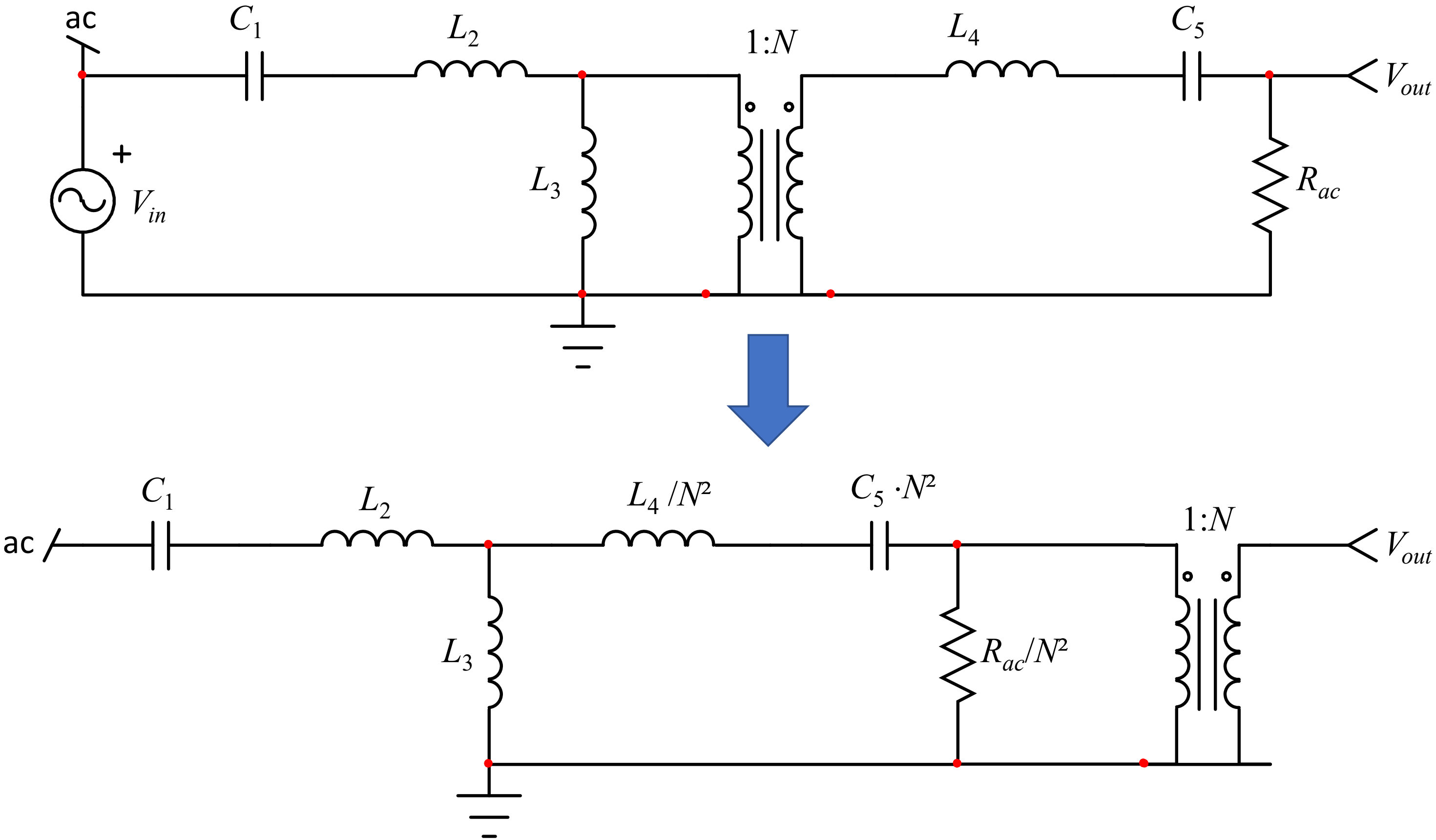
CLLC Transfer Function Determination





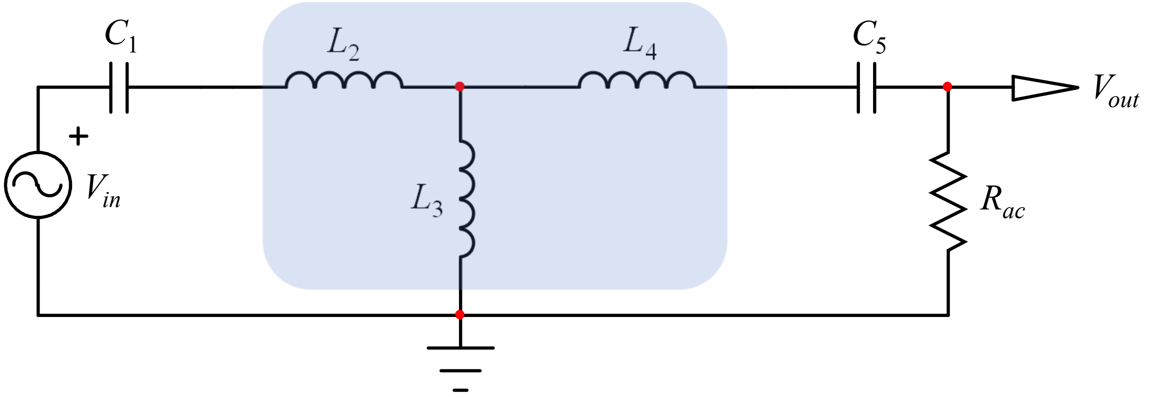
Christophe Basso
November 2022



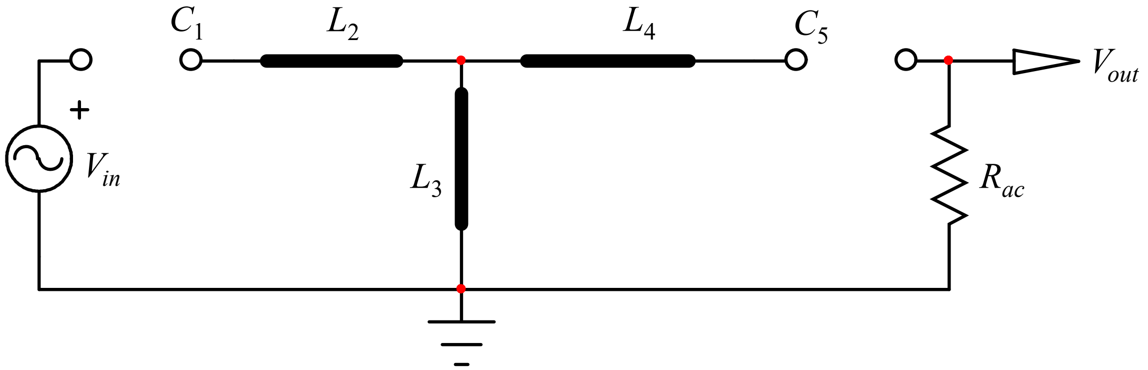
ac
C_1
L_2
1:N
L_4
C_5
V_{out}
+
V_{in}
L_3
R_{ac}
ac
C_1
L_2
L_4 /N^2
$C_5 \cdot N^2$
1:N
V_{out}
L_3
R_{ac}/N^2

Set $s = 0$

Degenerate case

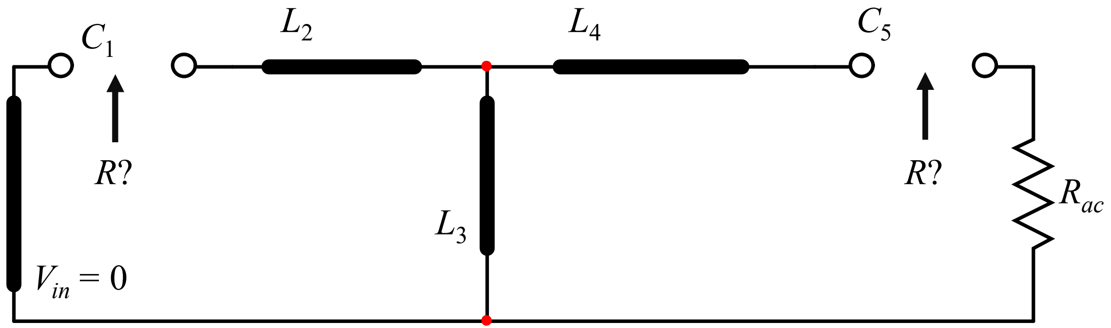


$H_0 = 0$



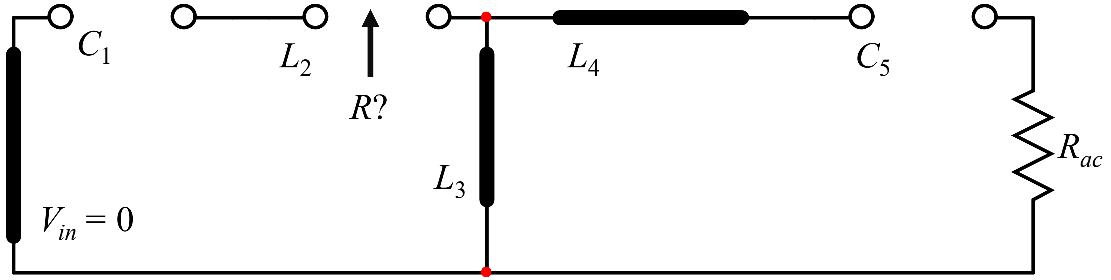
There are 5 energy-storing elements, this is a 4th-order TF because of the degenerate case (inductive node L_2-L_3-L_4)

Zero the excitation and determine time constants:

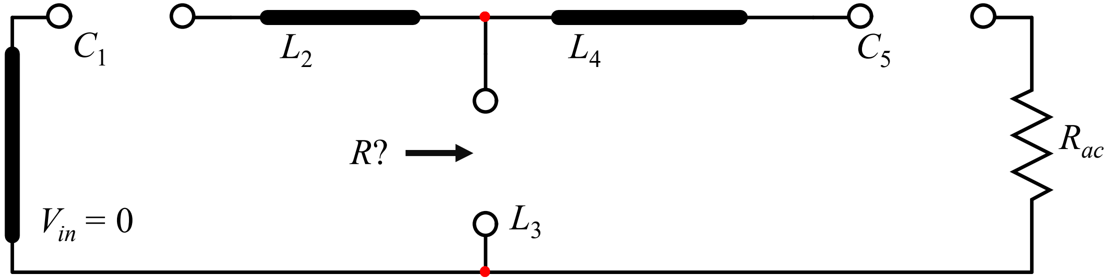


$$\tau_1 = R_s \cdot C_1$$

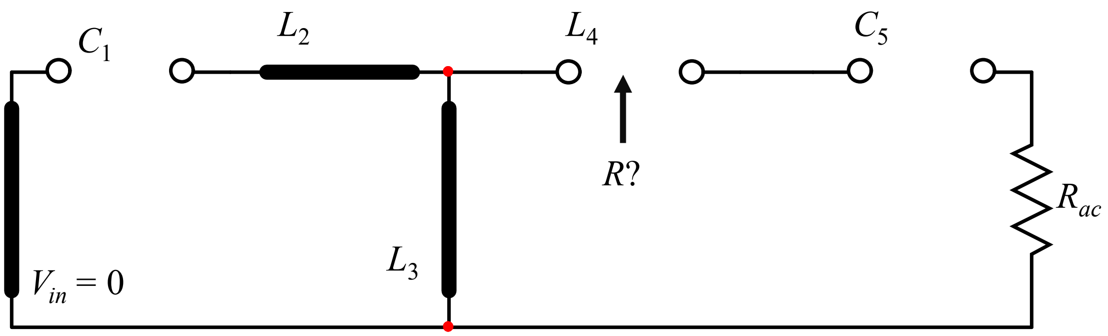
$$\tau_5 = R_{ac} \cdot C_5$$



$$\tau_2 = \frac{L_2}{R_{\inf}}$$



$$\tau_3 = \frac{L_3}{R_{\inf}}$$



$$\tau_4 = \frac{L_4}{R_{\inf}}$$

$$b_1 = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 = \underbrace{R_s C_1}_{=0} + R_{ac} C_5 + \underbrace{\frac{L_2}{R_{\inf}}}_{=0} + \underbrace{\frac{L_3}{R_{\inf}}}_{=0} + \underbrace{\frac{L_4}{R_{\inf}}}_{=0}$$

How many terms for b_2?

order → rank → $\binom{n}{j} = \frac{n!}{j!(n-j)!}$ ⇒ $\binom{5}{2} = 10$

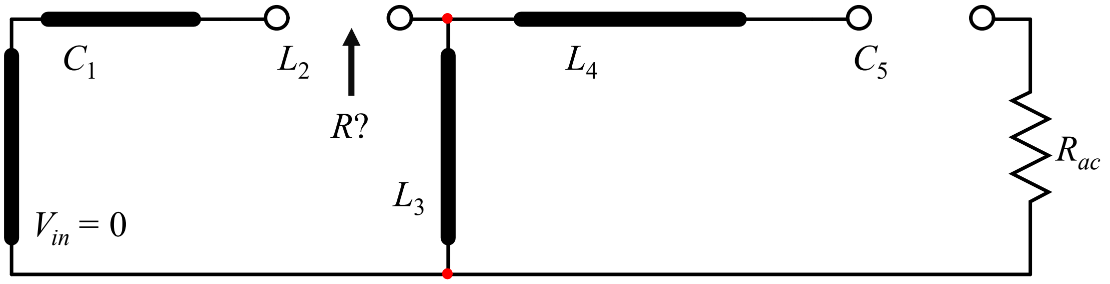
$$b_2 = \tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_1\tau_4^1 + \tau_1\tau_5^1 + \tau_2\tau_3^2 + \tau_2\tau_4^2 + \tau_2\tau_5^2 + \tau_3\tau_4^3 + \tau_3\tau_5^3 + \tau_4\tau_5^4$$

τ_2^1

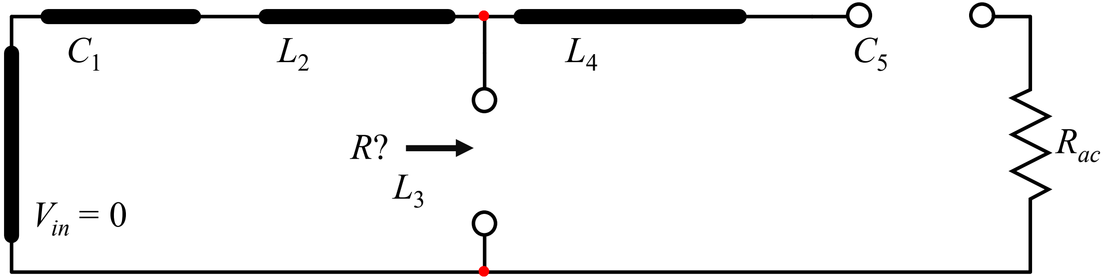
Set in high frequency
A short circuit for a cap.

$R\,?$

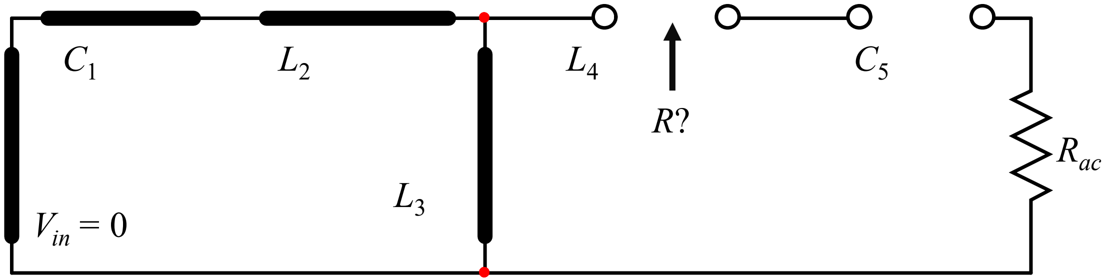
The remaining elements
are left in their dc state
(all caps open)



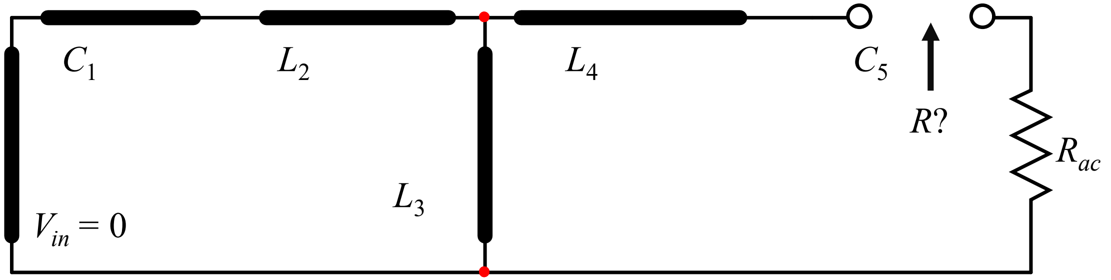
$$\tau_2^1 = \frac{L_2}{R_s}$$



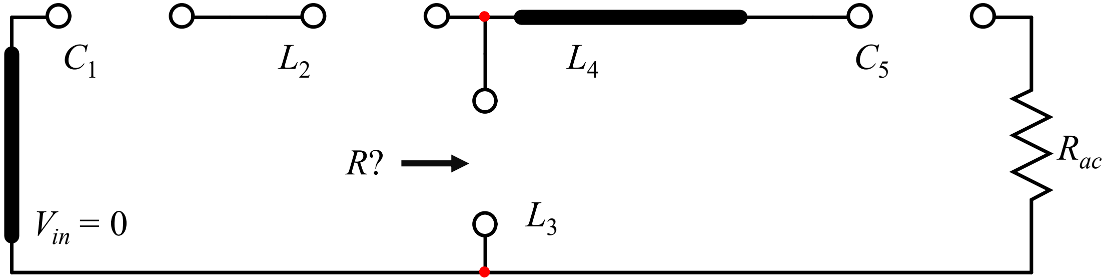
$$\tau_3^1 = \frac{L_3}{R_s}$$



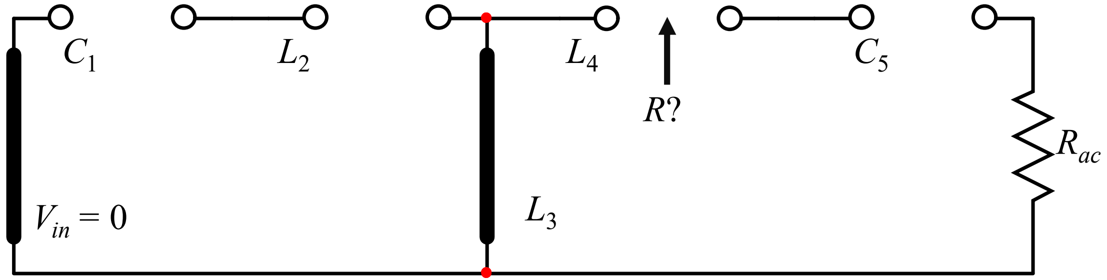
$$\tau_4^1 = \frac{L_4}{R_{\inf}}$$



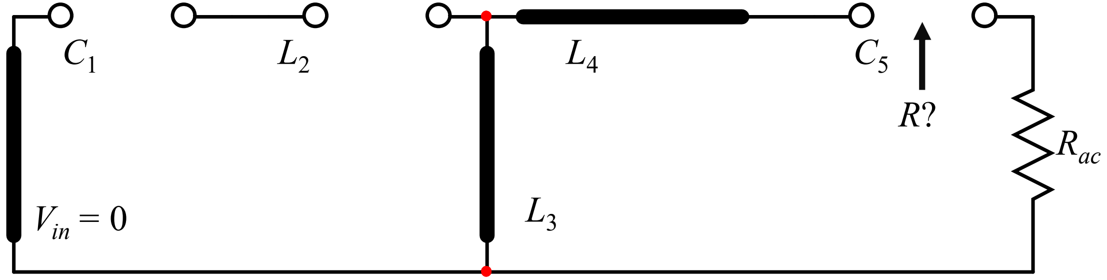
$$\tau_5^1 = C_5 R_{ac}$$



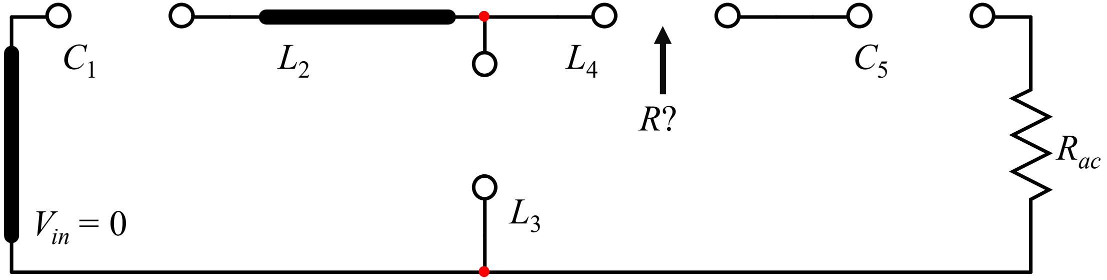
$$\tau_3^2 = \frac{L_3}{R_{\text{inf}}}$$



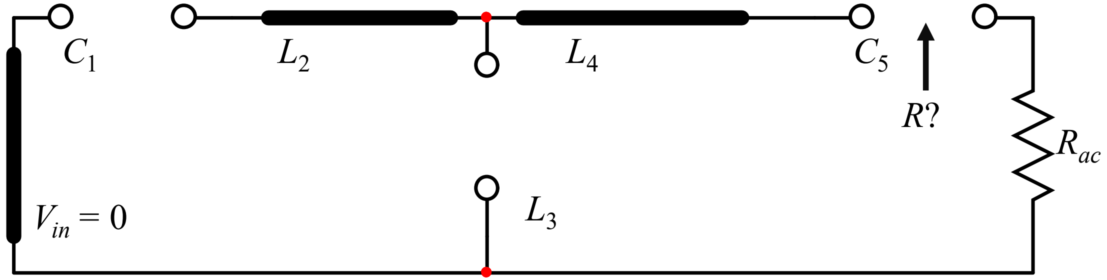
$$\tau_4^2 = \frac{L_4}{R_{\text{inf}}}$$



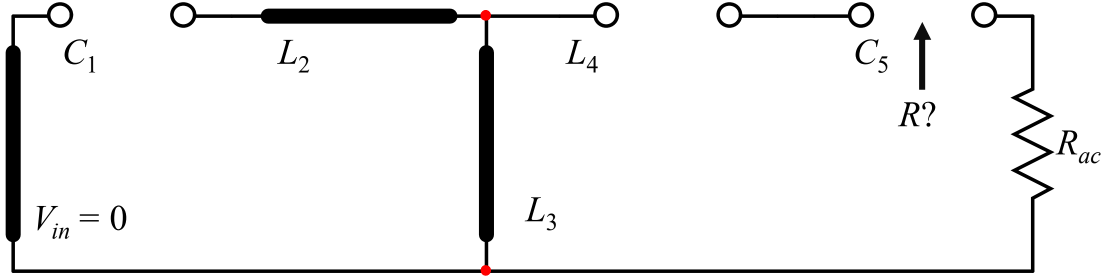
$$\tau_5^2 = C_5 R_{ac}$$



$$\tau_4^3 = \frac{L_4}{R_{\text{inf}}}$$



$$\tau_5^3 = C_5 R_{\text{inf}}$$



$$\tau_5^4 = C_5 R_{\text{inf}}$$

How many terms for b_3?

order → rank →

$$\binom{n}{j} = \frac{n!}{j!(n-j!)} \Rightarrow \binom{5}{3} = 10$$

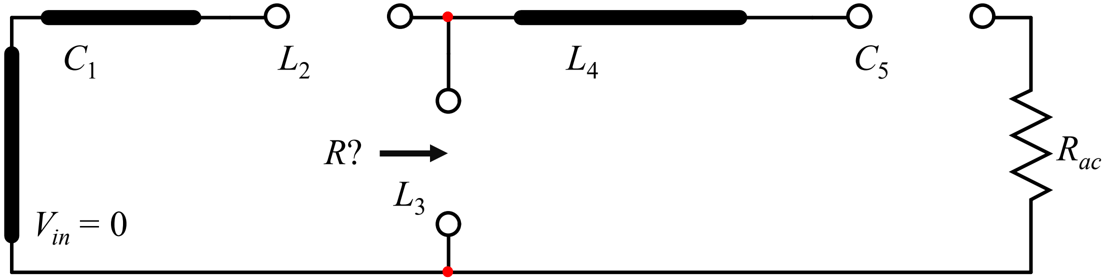
$$\tau_3^{12}$$

Set in high frequency
A short circuit for a cap.

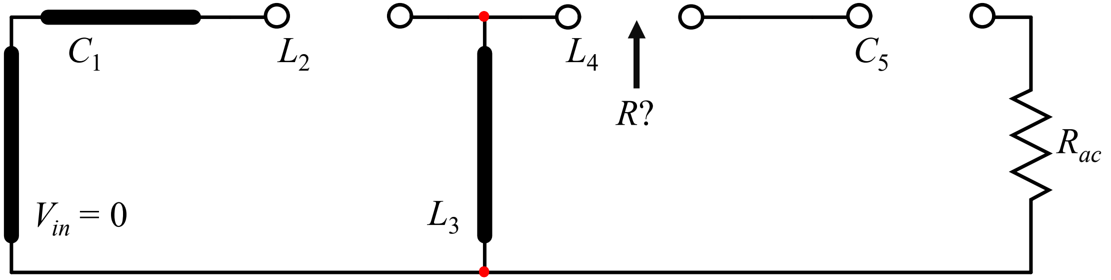
$R?$

The remaining elements are left in their dc state (all caps open)

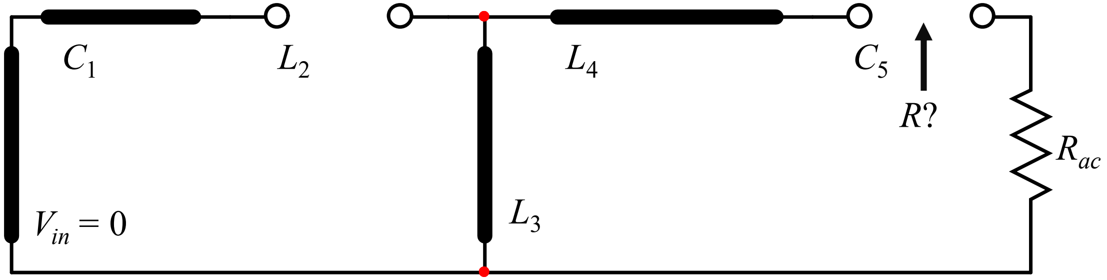
$$b_3 = \tau_1\tau_2^1\tau_3^{12} + \tau_1\tau_2^1\tau_4^{12} + \tau_1\tau_2^1\tau_5^{12} + \tau_1\tau_3^1\tau_4^{13} + \tau_1\tau_3^1\tau_5^{13} + \tau_1\tau_4^1\tau_5^{14} + \tau_2\tau_3^2\tau_4^{23} + \tau_2\tau_3^2\tau_5^{23} + \tau_2\tau_4^2\tau_5^{24} + \tau_3\tau_4^3\tau_5^{34}$$



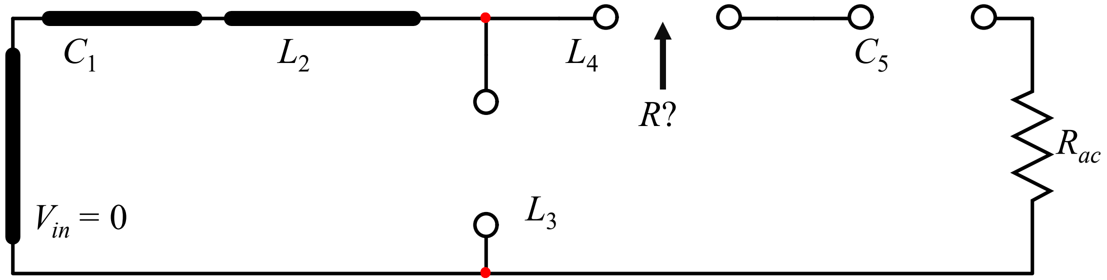
$$\tau_3^{12} = \frac{L_3}{R_{\inf}}$$



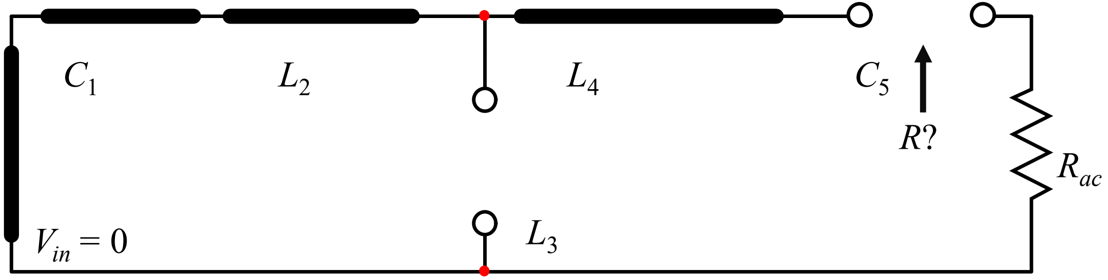
$$\tau_4^{12} = \frac{L_4}{R_{\inf}}$$



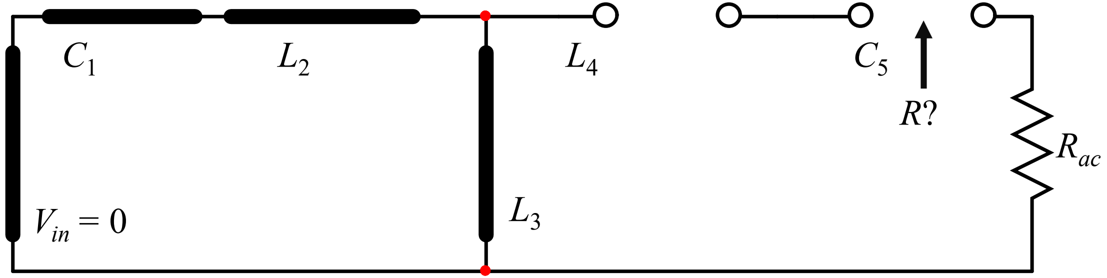
$$\tau_5^{12} = C_5 R_{ac}$$



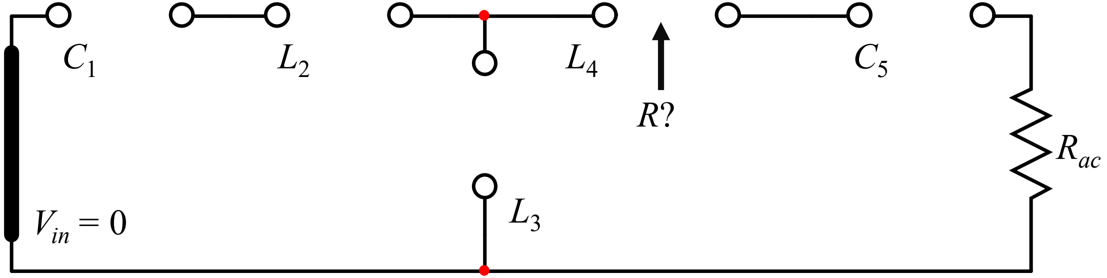
$$\tau_4^{13} = \frac{L_4}{R_{\inf}}$$



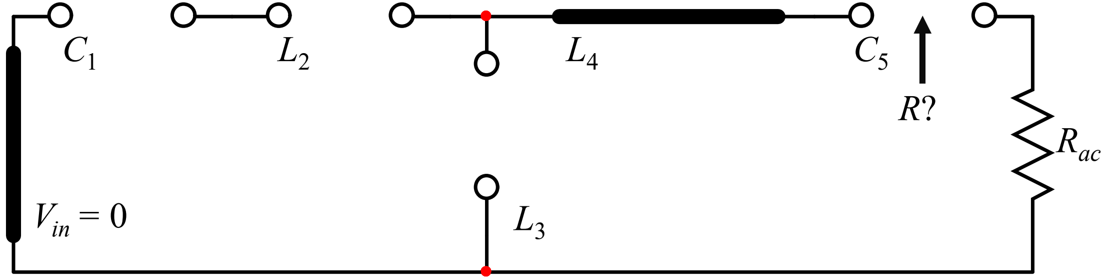
$$\tau_5^{13} = C_5 R_{ac}$$



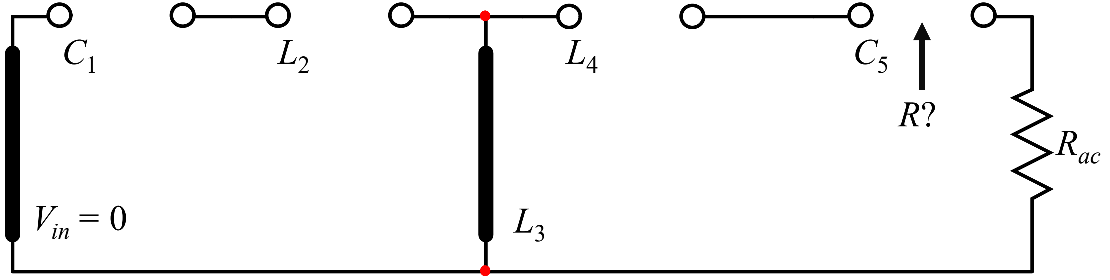
$$\tau_5^{14} = C_5 R_{\text{inf}}$$



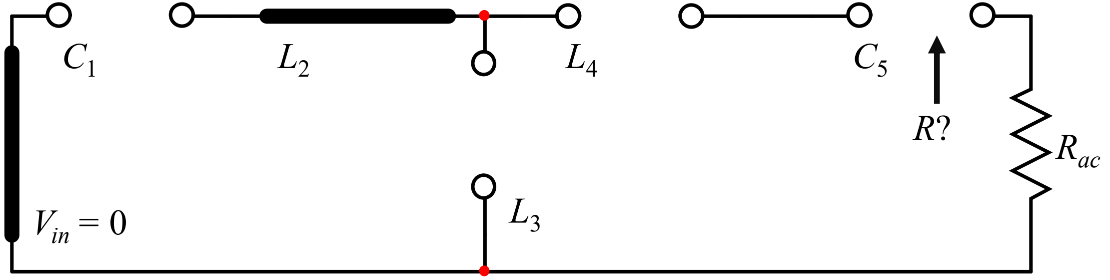
$$\tau_4^{23} = \frac{L_4}{R_{\text{inf}}}$$



$$\tau_5^{23} = C_5 R_{\text{inf}}$$



$$\tau_5^{24} = C_5 R_{\text{inf}}$$



$$\tau_5^{34} = C_5 R_{\text{inf}}$$

How many terms for b_4?

order → rank →
$$\binom{n}{j} = \frac{n!}{j!(n-j!)} \Rightarrow \binom{5}{4} = 5$$

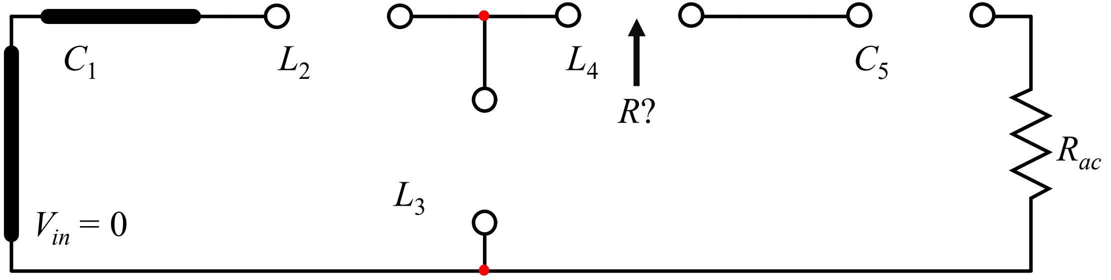
$$b_4 = \tau_1\tau_2^1\tau_3^{12}\tau_4^{123} + \tau_1\tau_2^1\tau_3^{12}\tau_5^{123} + \tau_1\tau_2^1\tau_4^{12}\tau_5^{124} + \tau_1\tau_3^1\tau_4^{13}\tau_5^{134} + \tau_2\tau_3^2\tau_4^{23}\tau_5^{234}$$

$$\tau_4^{123}$$

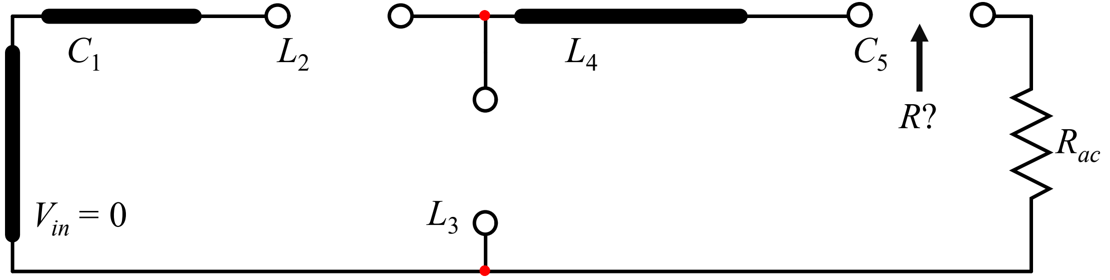
123 ← Set in high frequency
A short circuit for a cap.

4 ← R?

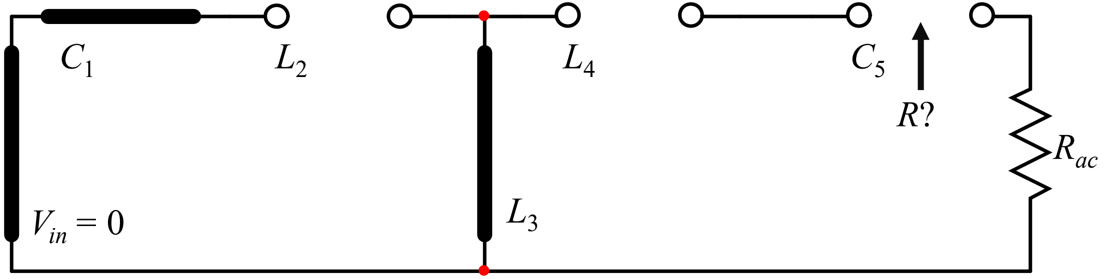
The remaining elements are left in their dc state (all caps open)



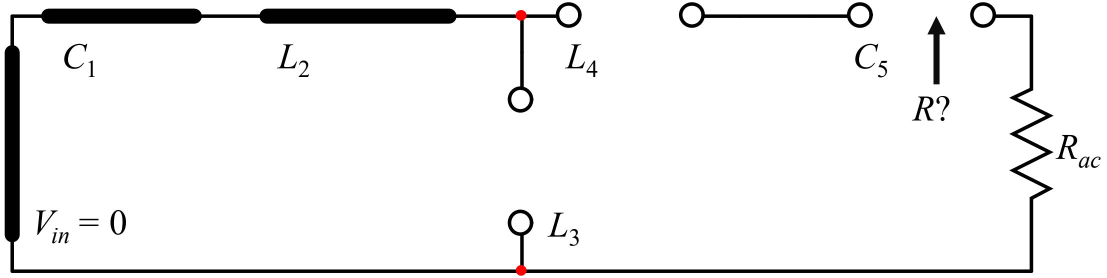
$$\tau_4^{123} = \frac{L_4}{R_{\mathrm{inf}}}$$



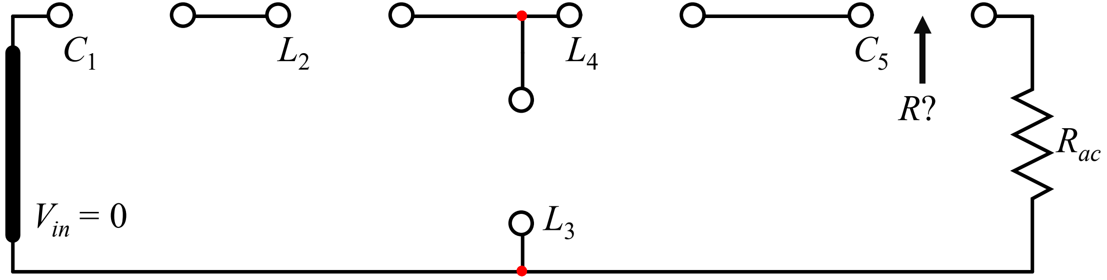
$$\tau_5^{123} = C_5 R_{\mathrm{inf}}$$



$$\tau_5^{124} = C_5 R_{\inf}$$



$$\tau_5^{134} = C_5 R_{\inf}$$



$$\tau_5^{234} = C_5 R_{\inf}$$

How many terms for b_5?

order → rank →
$$\binom{n}{j} = \frac{n!}{j!(n-j!)} \Rightarrow \binom{5}{5} = 1$$

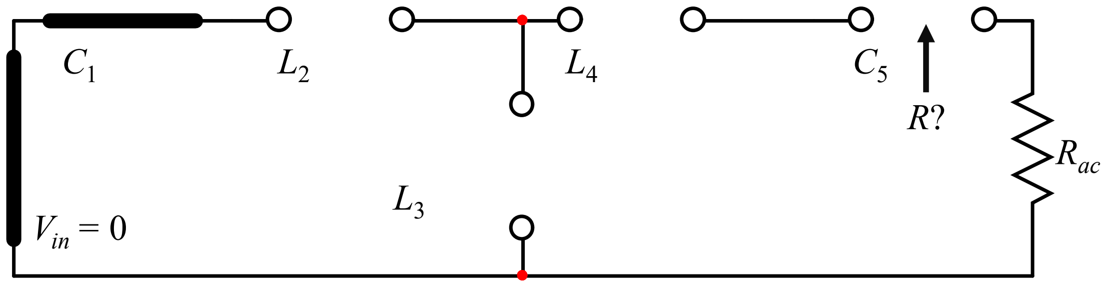
$$b_5 = \tau_1 \tau_2^1 \tau_3^{12} \tau_4^{123} \tau_5^{1234}$$

$$\tau_5^{1234}$$

τ_5^{1234} ← Set in high frequency
A short circuit for a cap.
An open circuit for *L*

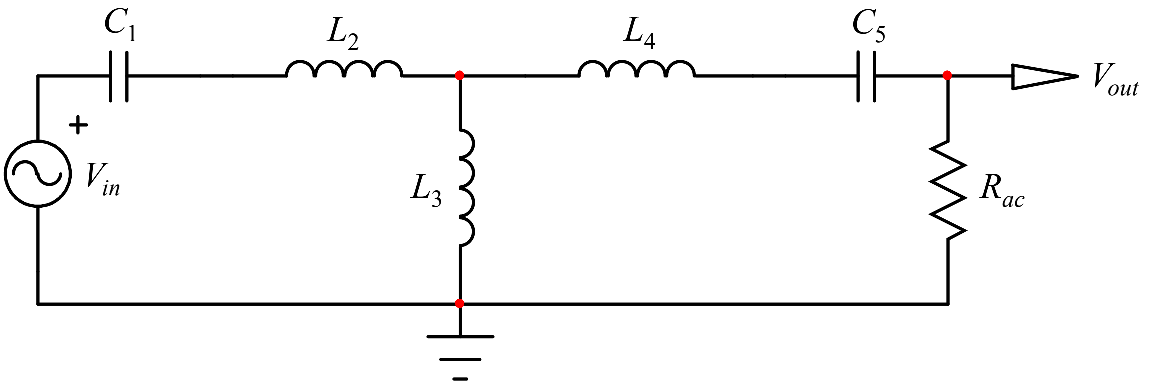
↑ $R?$

The remaining elements are left in their dc state (all caps open, all shorted)



$$\tau_5^{1234} = C_5 R_{\text{inf}}$$

Determining high-frequency gains



The only combination in which a gain exists is when C_1, L_3 and C_5 are in their high-frequency state: $H^{135} = 1$

H^{1} ← Element 1 is in HF. The rest are in dc. What is the gain?

$H^1 = 0$

$H^2 = 0$

$H^3 = 0$

$H^4 = 0$

$H^5 = 0$

H^{12} ← Element 12 are in HF. The rest are in dc. What is the gain?

$H^{12} = 0$ $H^{25} = 0$

$H^{13} = 0$ $H^{34} = 0$

$H^{14} = 0$ $H^{35} = 0$

$H^{15} = 0$ $H^{45} = 0$

$H^{23} = 0$

$H^{24} = 0$

H^{123} ← Element 123 are in HF. The rest are in dc. What is the gain?

$H^{123} = 0$ $H^{234} = 0$

$H^{124} = 0$ $H^{235} = 0$

$H^{125} = 0$ $H^{245} = 0$

$H^{134} = 0$ $H^{345} = 0$

$H^{135} = 1$

$H^{145} = 0$

H^{1234} ← Element 1234 are in HF. The rest are in dc. What is the gain?

$H^{1234} = 0$

$H^{1235} = 0$

$H^{1245} = 0$

$H^{1345} = 0$

$H^{2345} = 0$

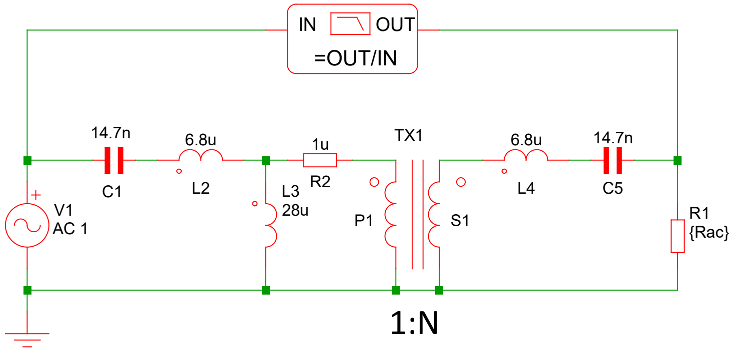
$H^{12345} = 0$

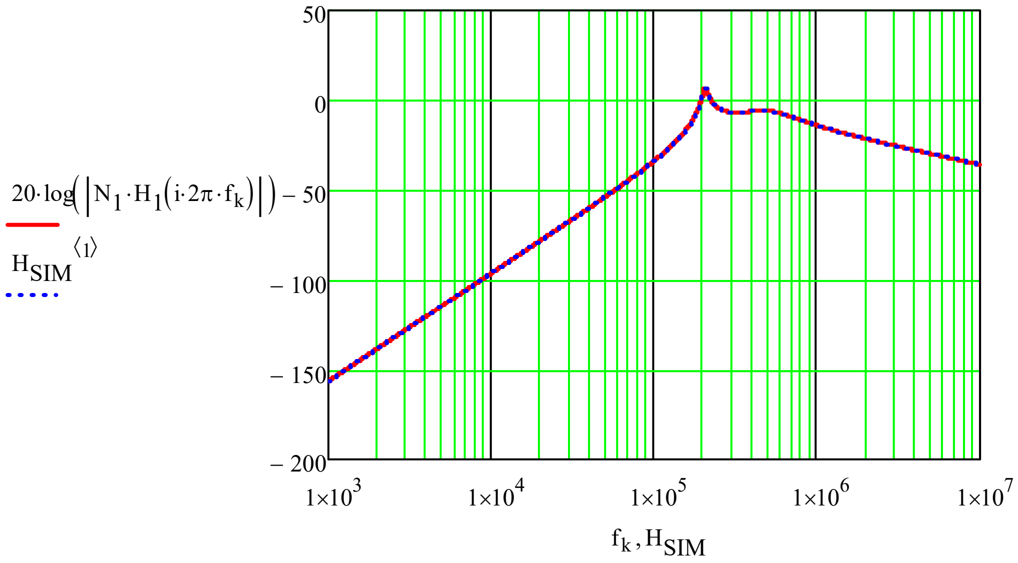
$$H(s) = \frac{H_0 + s^3 H^{135}\tau_1\tau_3^1\tau_5^{13}}{D(s)} = \frac{s^3\tau_1\tau_3^1\tau_5^{13}}{1+sb_1+s^2b_2+s^3b_3+s^4b_4+s^5\underbrace{b_5}_{=0}}$$

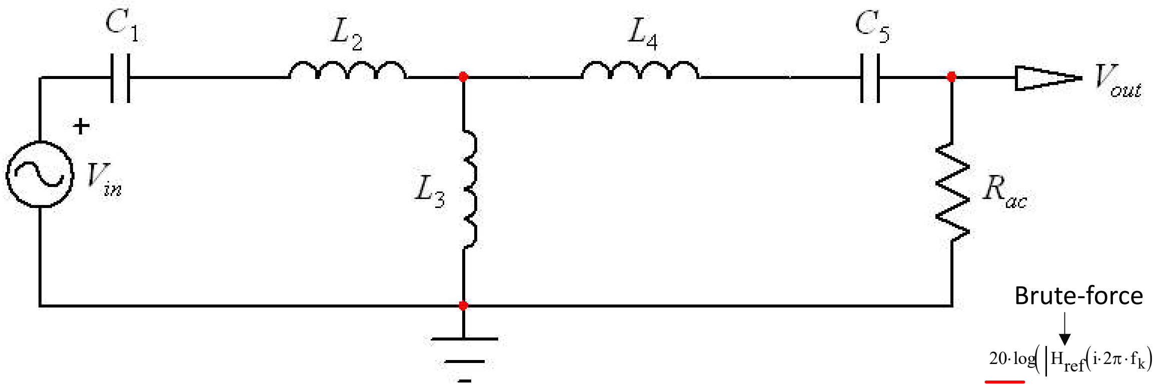
Turns ratio, 1:N

$$H(s) = N\frac{s^3 R_{ac} L_3 C_5 C_1}{1+sR_{ac}C_5+s^2\left[C_1\left(L_2+L_3\right)+C_5\left(L_3+L_4\right)\right]+s^3 C_1 C_5 R_{ac}\left(L_2+L_3\right)+s^4 C_1 C_5\left(L_2L_3+L_2L_4+L_3L_4\right)}$$

```
*
.PARAM Pout=10k
.PARAM Vout=500
.PARAM RL={Vout^2/Pout}
.PARAM Rac={8*RL/pi^2}
*
```



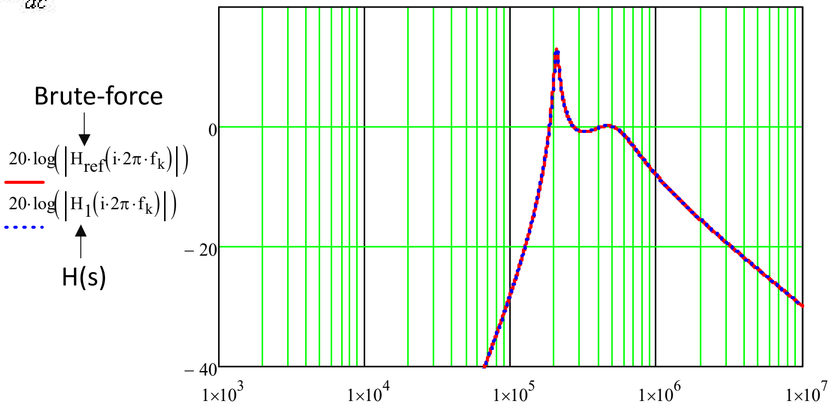




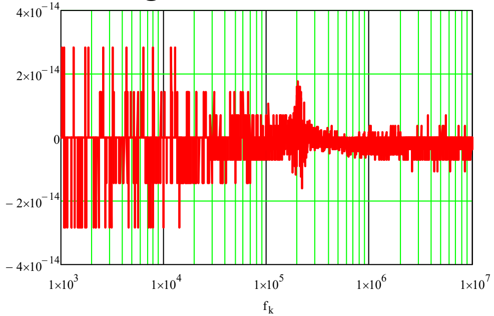
$$Z_1(s) := \frac{1}{s \cdot C_1} + s \cdot L_2 \qquad Z_4(s) := s \cdot L_4 + \frac{1}{s \cdot C_5}$$

$$H_{ref}(s) := \frac{s \cdot L_3}{s \cdot L_3 + Z_1(s)} \cdot \frac{R_{ac}}{Z_4(s) + R_{ac} + (s \cdot L_3) \parallel Z_1(s)}$$

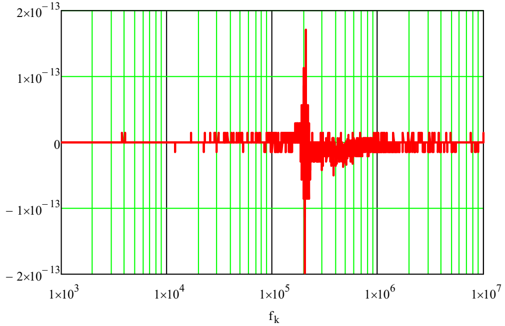
Brute force transfer function

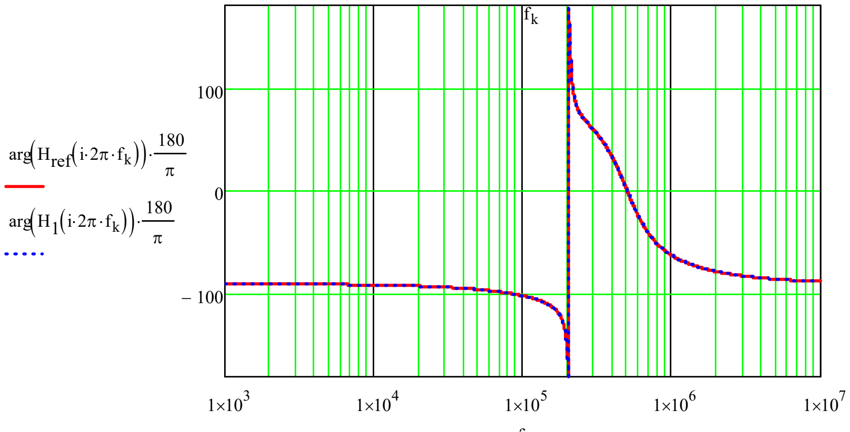


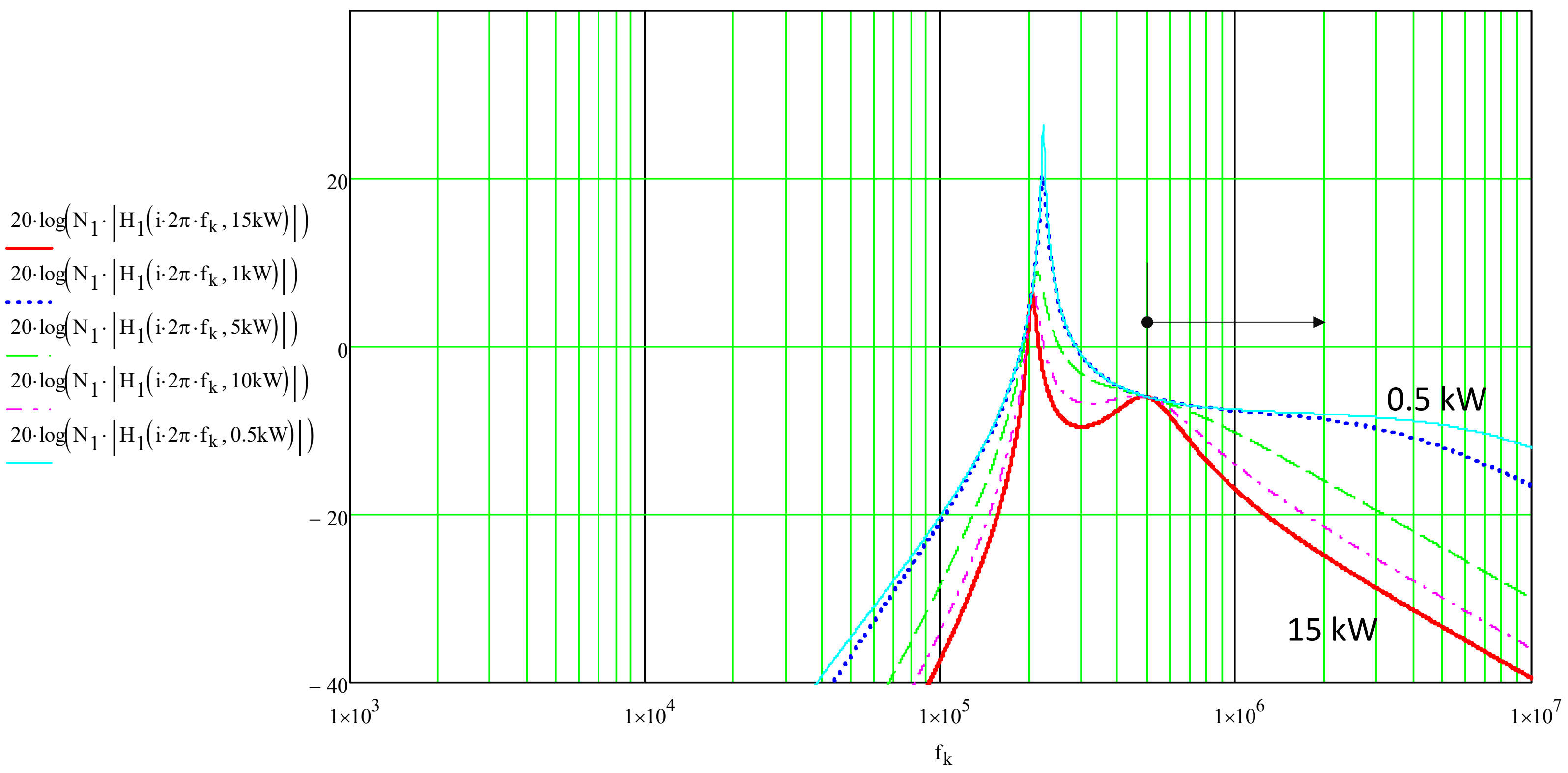
Magnitude difference



Phase difference






20·log(N1·|H1(i·2π·fk, 15kW)|)
20·log(N1·|H1(i·2π·fk, 1kW)|)
20·log(N1·|H1(i·2π·fk, 5kW)|)
20·log(N1·|H1(i·2π·fk, 10kW)|)
20·log(N1·|H1(i·2π·fk, 0.5kW)|)
20
0
– 20
– 40
1×10^3
1×10^4
1×10^5
1×10^6
1×10^7
fk
0.5 kW
15 kW

$\|(x,y) := \frac{x \cdot y}{x+y}$

CLLC converter transfer function

$bino(n,j) := \frac{n!}{j! \cdot (n-j)!}$

$C_1 := 14.7nF \quad N_1 := 0.5 \quad L_2 := 6.8\mu H \quad L_3 := 28\mu H$

turns ratio N1 is 1:n

$P_{out} := 10kW \quad V_{out} := 500V \quad R_L := \frac{V_{out}^2}{P_{out}} = 25\Omega$

$R_{ac} := \frac{8}{\pi^2} \cdot \frac{R_L}{N_1^2} = 81.0569\Omega$

$C_5 := C_1 \cdot N_1^2 = 3.675nF \quad L_4 := \frac{L_2}{N_1^2} = 27.2\mu H$

$R_{inf} := 10^{23}\Omega \quad R_s := 10^{-23}\Omega$

$H_0 := 0$

$\tau_1 := R_s \cdot C_1 = 0 \cdot \mu s \quad \tau_2 := \frac{L_2}{R_{inf}} = 0 \cdot \mu s \quad \tau_3 := \frac{L_3}{R_{inf}} = 0 \cdot \mu s$

$\tau_4 := \frac{L_4}{R_{inf}} = 0 \cdot \mu s \quad \tau_5 := R_{ac} \cdot C_5 = 2.9788 \times 10^{-7} s$

$b_1 := \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 = 0.2979\mu s$

How many terms in b2?

$bino(5,2) = 10$ 5th order, term 2

$\tau_{12} := \frac{L_2}{R_s} = 6.8 \times 10^{17} s \quad \tau_{13} := \frac{L_3}{R_s} = 2.8 \times 10^{18} s$

$\tau_{14} := \frac{L_4}{R_{inf}} = 0s \quad \tau_{15} := R_{ac} \cdot C_5 = 2.9788 \times 10^{-7} s$

$\tau_{23} := \frac{L_3}{R_{inf}} = 0s \quad \tau_{24} := \frac{L_4}{R_{inf}}$

$\tau_{25} := R_{ac} \cdot C_5 = 2.9788 \times 10^{-7} s \quad \tau_{34} := \frac{L_4}{R_{inf}} = 0s$

$\tau_{35} := R_{inf} \cdot C_5 = 3.675 \times 10^{14} s \quad \tau_{45} := R_{inf} \cdot C_5 = 3.675 \times 10^{14} s$

$$b_2 := \tau_1 \cdot \tau_{12} + \tau_1 \cdot \tau_{13} + \tau_1 \cdot \tau_{14} + \tau_1 \cdot \tau_{15} + \tau_2 \cdot \tau_{23} + \tau_2 \cdot \tau_{24} + \tau_2 \cdot \tau_{25} + \tau_3 \cdot \tau_{34} + \tau_3 \cdot \tau_{35} + \tau_4 \cdot \tau_{45} = 7.1442 \times 10^{-13} s^2$$

$$b_2 := R_s \cdot C_1 \cdot \frac{L_2}{R_s} + R_s \cdot C_1 \cdot \frac{L_3}{R_s} + R_s \cdot C_1 \cdot \frac{L_4}{R_{inf}} + R_s \cdot C_1 \cdot (R_{ac} \cdot C_5) + \frac{L_2}{R_{inf}} \cdot \frac{L_3}{R_{inf}} + \frac{L_2}{R_{inf}} \cdot \frac{L_4}{R_{inf}} + \frac{L_2}{R_{inf}} \cdot (R_{ac} \cdot C_5) + \frac{L_3}{R_{inf}} \cdot \frac{L_4}{R_{inf}} + \frac{L_3}{R_{inf}} \cdot (R_{inf} \cdot C_5) + \frac{L_4}{R_{inf}} \cdot (R_{inf} \cdot C_5) = 7.1442 \times 10^{-13} s^2$$

$$b_2 := C_1 \cdot L_2 + C_1 \cdot L_3 + 0 + 0 + 0 + 0 + 0 + 0 + L_3 \cdot C_5 + L_4 \cdot C_5 = 7.1442 \times 10^{-13} s^2$$

How many terms in b3?

$\mathrm{bino}(5,3) = 10$ 5th order, term 3

$\tau_{123} := \frac{L_3}{R_{inf}} = 0\,s$ $\tau_{125} := R_{ac} \cdot C_5 = 2.9788\times 10^{-7}\,s$

$\tau_{124} := \frac{L_4}{R_{inf}} = 0\,s$ $\tau_{134} := \frac{L_4}{R_{inf}} = 0\,s$

$\tau_{135} := R_{ac} \cdot C_5 = 2.9788\times 10^{-7}\,s$ $\tau_{145} := R_{inf} \cdot C_5 = 3.675\times 10^{14}\,s$

$\tau_{234} := \frac{L_4}{R_{inf}} = 0\,s$ $\tau_{235} := R_{inf} \cdot C_5 = 3.675\times 10^{14}\,s$

$\tau_{245} := R_{inf} \cdot C_5 = 3.675\times 10^{14}\,s$ $\tau_{345} := R_{inf} \cdot C_5 = 3.675\times 10^{14}\,s$

$$b_3 := \tau_1 \cdot \tau_{12} \cdot \tau_{123} + \tau_1 \cdot \tau_{12} \cdot \tau_{124} + \tau_1 \cdot \tau_{12} \cdot \tau_{125} + \tau_1 \cdot \tau_{13} \cdot \tau_{134} + \tau_1 \cdot \tau_{13} \cdot \tau_{135} + \tau_1 \cdot \tau_{14} \cdot \tau_{145} + \tau_2 \cdot \tau_{23} \cdot \tau_{234} + \tau_2 \cdot \tau_{23} \cdot \tau_{235} + \tau_2 \cdot \tau_{24} \cdot \tau_{245} + \tau_3 \cdot \tau_{34} \cdot \tau_{345} = 1.5239\times 10^{8} \cdot ns^3$$

$$b_3 := R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot \frac{L_3}{R_{inf}} + R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot \frac{L_4}{R_{inf}} + R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot (R_{ac} \cdot C_5) + R_s \cdot C_1 \cdot \frac{L_3}{R_s} \cdot \frac{L_4}{R_{inf}} + R_s \cdot C_1 \cdot \frac{L_3}{R_s} \cdot (R_{ac} \cdot C_5) + R_s \cdot C_1 \cdot \frac{L_4}{R_{inf}} \cdot (R_{inf} \cdot C_5) + \frac{L_2}{R_{inf}} \cdot \frac{L_3}{R_{inf}} \cdot \frac{L_4}{R_{inf}} + \frac{L_2}{R_{inf}} \cdot \frac{L_3}{R_{inf}} \cdot (R_{inf} \cdot C_5) + \frac{L_2}{R_{inf}} \cdot \frac{L_4}{R_{inf}} \cdot (R_{inf} \cdot C_5) + \frac{L_3}{R_{inf}} \cdot \frac{L_4}{R_{inf}} \cdot (R_{inf} \cdot C_5) = 1.5239\times 10^{8} \cdot ns^3$$

$$b_3 := 0 + 0 + C_1 \cdot L_2 \cdot C_5 \cdot R_{ac} + 0 + C_1 \cdot L_3 \cdot R_{ac} \cdot C_5 + 0 + 0 + 0 + 0 + 0 = 1.5239\times 10^{8} \cdot ns^3$$

How many terms in b4?

$\mathrm{bino}(5,4) = 5$ 5th order, term 4

$\tau_{1234} := \frac{L_4}{R_{inf}} = 0\,s$

$\tau_{1245} := C_5 \cdot R_{inf} = 3.675\times 10^{14}\,s$

$\tau_{1235} := C_5 \cdot R_{inf} = 3.675\times 10^{14}\,s$

$\tau_{1345} := C_5 \cdot R_{inf} = 3.675\times 10^{14}\,s$

$\tau_{2345} := C_5 \cdot R_{inf} = 3.675\times 10^{14}\,s$

$$b_4 := \tau_1 \cdot \tau_{12} \cdot \tau_{123} \cdot \tau_{1234} + \tau_1 \cdot \tau_{12} \cdot \tau_{123} \cdot \tau_{1235} + \left(\tau_1 \cdot \tau_{12} \cdot \tau_{124} \cdot \tau_{1245} + \tau_1 \cdot \tau_{13} \cdot \tau_{134} \cdot \tau_{1345} + \tau_2 \cdot \tau_{23} \cdot \tau_{234} \cdot \tau_{2345}\right) = 6.1421\times 10^{10} \cdot ns^4$$

$$b_4 := R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot \frac{L_3}{R_{inf}} \cdot \frac{L_4}{R_{inf}} + R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot \frac{L_3}{R_{inf}} \cdot (C_5 \cdot R_{inf}) + \left[R_s \cdot C_1 \cdot \frac{L_2}{R_s} \cdot \frac{L_4}{R_{inf}} \cdot (C_5 \cdot R_{inf}) + R_s \cdot C_1 \cdot \frac{L_3}{R_s} \cdot \frac{L_4}{R_{inf}} \cdot (C_5 \cdot R_{inf}) + \frac{L_2}{R_{inf}} \cdot \frac{L_3}{R_{inf}} \cdot \frac{L_4}{R_{inf}} \cdot (C_5 \cdot R_{inf})\right] = 6.1421\times 10^{10} \cdot ns^4$$

$$b_4 := 0 + C_1 \cdot C_5 \cdot L_2 \cdot L_3 + \left(C_1 \cdot C_5 \cdot L_2 \cdot L_4 + C_1 \cdot L_3 \cdot L_4 \cdot C_5 + 0\right) = 6.1421\times 10^{10} \cdot ns^4$$

How many terms in b5?

$\text{bino}(5,5) = 1$ 5th order, term 5

$\tau_{12345} := C_5 \cdot R_{inf} = 3.675\times 10^{14}\,s$

$b_5 := \tau_1 \cdot \tau_{12} \cdot \tau_{123} \cdot \tau_{1234} \cdot \tau_{12345} = 2.7978\times 10^{-9} \cdot ns^5$

$b_5 := 1 \cdot C_1 \cdot \frac{L_2}{1} \cdot \frac{L_3}{1} \cdot \frac{L_4}{R_{inf}} \cdot (C_5 \cdot 1) = 2.7978\times 10^{-9} \cdot ns^5$

b5 is actually zero because
of the degenerate case

$$H_1(s) := \frac{s^3 \cdot \tau_1 \cdot \tau_{13} \cdot \tau_{135}}{1 + b_1 \cdot s + b_2 \cdot s^2 + b_3 \cdot s^3 + b_4 \cdot s^4 + 0 \cdot s^5}$$

$$H_1(s) := \frac{s^3 \cdot (R_{ac} \cdot L_3 \cdot C_5 \cdot C_1)}{1 + R_{ac} \cdot C_5 \cdot s + [C_1 \cdot (L_2 + L_3) + C_5 \cdot (L_3 + L_4)] \cdot s^2 + [C_1 \cdot C_5 \cdot R_{ac} \cdot (L_2 + L_3)] \cdot s^3 + [C_1 \cdot C_5 \cdot (L_2 \cdot L_3 + L_2 \cdot L_4 + L_3 \cdot L_4)] \cdot s^4}$$

Time constants studies:

$b_1 = 297.8843\,ns$ $\frac{b_2}{b_1} = 2.3983\,\mu s$ $\frac{b_3}{b_2} = 213.2999\,ns$ $\frac{b_4}{b_3} = 403.0656\,ns$

$\omega_{01} := \frac{1}{\sqrt{b_2}}$ $f_{01} := \frac{\omega_{01}}{2 \cdot \pi} = 188.297\,kHz$

$\omega_{02} := \frac{1}{\omega_{01} \cdot \sqrt{b_4}} = 3.4105\times 10^6 \frac{1}{s}$ $f_{02} := \frac{\omega_{02}}{2 \cdot \pi} = 542.7965\,kHz$

$Q_1 := \frac{1}{b_1 \cdot \omega_{01}} = 2.8375$ $Q_2 := \frac{\omega_{02}}{\frac{b_3}{b_4} - \frac{\omega_{01}}{Q_1}} = 1.6523$

$$H_2(s) := \frac{\left[s^3 \cdot (R_{ac} \cdot L_3 \cdot C_5 \cdot C_1)\right]}{\left[1 + \frac{s}{\omega_{01} \cdot Q_1} + \left(\frac{s}{\omega_{01}}\right)^2\right] \cdot \left[1 + \frac{s}{\omega_{02} \cdot Q_2} + \left(\frac{s}{\omega_{02}}\right)^2\right]}$$

$H_{SIM} := \text{READPRN}(\text{".\textbackslash TF.txt"})$