Designing Compensators for the Control of Switching Power Supplies

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Agenda

- Feedback generalities
- The divider and the virtual ground
- Phase margin and crossover
- Poles and zeros
- Boosting the phase at crossover
- Compensator types
- Practical implementations: the op amp
- Practical implementations: the OTA
- Practical implementations: the TL431
- Design examples
- A real case study
- Conclusion
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What is a regulated power supply?

- $V_{\text{out}}$ is permanently compared to a reference voltage $V_{\text{ref}}$.
- The reference voltage $V_{\text{ref}}$ is precise and stable over temperature.
- The error $\varepsilon = V_{\text{ref}} - \alpha V_{\text{out}}$ is amplified and sent to the control input.
- The power stage reacts to reduce $\varepsilon$ as much as it can.
How to build an oscillator?

- How to keep self-sustained oscillations?

\[
V_{in}(s) \xrightarrow{+} \quad H(s) \quad \xrightarrow{-} \quad V_{out}(s)
\]

\[
V_{out}(s) = \frac{H(s)}{1 + H(s)G(s)} \quad \text{Open-loop gain } T(s)
\]

\[
V_{out}(s) = \lim_{s \to 0} \left[ \frac{H(s)}{1 + G(s)H(s)}V_{in}(s) \right]
\]

To sustain self-oscillations, as \( V_{in}(s) \) goes to zero, quotient must go infinite.

\[
1 + G(s)H(s) = 0 \quad \Rightarrow \quad T(s) = \begin{cases} 
G(s)H(s) = 1 = 0 \text{ dB} \\
\angle G(s)H(s) = -180°
\end{cases}
\]

Where is the point \(-1,j0\)?

- In a Bode plot, we deal with both magnitude and argument:
  - when \(|T(s)|\) crosses the 0-dB axis, this is the "1" point
  - when \(\angle T(s)\) crosses the -180° axis, this is the "-" sign

- In a Nyquist plot, we deal with the argument and real part of \(T(s)\)
  - the point \(-1,j0\) represents the 0-dB gain and the sign reversal
If you fear oscillations, build phase margin!

- The frequency at which $|T(s)| = 0$ dB is the crossover frequency, $f_c$
- The distance between arg $T(f_c)$ and the -180° limit is called:
  - the phase margin, noted $\phi_m$

$$f_c = 6.5\ \text{kHz}$$

$$\phi_m = 92°$$

Gain margin 67 dB

In the literature, $V_{out}$ must follow $V_{in}$

- Text books cover loop control theory assuming $V_{out}$ follows $V_{in}$:
  - If $V_{in}$ imposes a ramp, $V_{out}$ must follow with the least error
  - The loop is then open to check $V_{out}$ over $V_{in}$

In our converters, $V_{in}$ is $V_{ref}/\alpha$ and is fixed!
- If the loop gain is high enough, we should have: $V_{out} = V_{ref}/\alpha$
- The perturbations are $V_{in}$ and $I_{out}$
  - The model must be updated
How does this translate to our converter?

- The loop gain $T(s)$ includes $H(s)$, $G(s)$ and $G_{PWM}$

$$V_{ref} / \alpha \rightarrow G_{PWM} \rightarrow H(s) \rightarrow G(s) \rightarrow V_{out}(s)$$

$$T(s) = G_{PWM} H(s) G(s)$$

- In the literature, $T(s)$ is considered without the phase reversal brought by the negative feedback:

$$\angle T(s) = -180^\circ$$ brings instability

In real life, we include the phase reversal!

$$V_{ref} / \alpha \rightarrow G_{PWM} \rightarrow H(s) \rightarrow G(s) \rightarrow V_{out}(s)$$

$$T(s) = -G_{PWM} H(s) G(s)$$

- In the real life, $T(s)$ includes the phase reversal brought by the negative feedback:

$$\angle T(s) = -360^\circ$$ brings instability
These plots are identical

- all these plots read the same phase margin!

Gain and phase plots for SPICE Network analyzer and Literature.

0°: modulo 360° (or modulo 2π) reading

-180°: power stage \( H(s) \), comp. \( G(s) \)

-360° is a complete turn!

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Hey, where is the divider network?

- In some textbooks, the divider network enters the picture

$$\frac{V_c(s)}{V_{out}(s)} = \frac{R_{lower}}{R_{lower} + R_{upper}} G(s)$$


The virtual ground excludes $R_{lower}$

- In reality, the feedback is often made with an op amp

$$V_c(s) = \frac{Z_f}{V_{out}(s)}$$

- Because of the local feedback via $Z_f$, we have a virtual ground
Looks like the divider in back…

- In a type 1, 2 or 3, the local feedback is lost for $s = 0$

$$V_{out}(s)$$

$$V_c(s)$$

$$V_{ref}$$

$$V_{out}(0) = -\frac{R_{lower}}{R_{lower} + R_{upper}} A$$

- The 0-Hz gain is indeed changed but not $f_c$!

Looks like the divider in back…

- With an op amp, only the dc gain is affected

$$\alpha = 0.5$$

$$\alpha = 0.2$$

$$A_v = 60 \text{ dB}$$
You don’t have a virtual ground in an OTA!

- $R_{\text{lower}}$ enters the picture in all equations

\[ R_{\text{eq}} = \frac{R_{\text{upper}} + R_{\text{lower}}}{R_{\text{lower}}} \cdot gm \]

- Unless an OTA is used, the divider plays no role in ac!

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How much phase margin to choose?

- A Q factor of 0.5 (critical response) implies a $\phi_m$ of 76°.
- A 45° $\phi_m$ corresponds to a Q of 1:2: oscillatory response!

- Phase margin depends on the needed response: fast, no overshoot...
- Good practice is to shoot for 60° and make sure $\phi_m$ always > 45°.

Which crossover frequency to select?

- Crossover frequency selection depends on several factors:
  - switching frequency: theoretical limit is $F_{sw}/2$
    - In practice, stay below 1/5 of $F_{sw}$ for noise concerns
  - output ripple: if ripple pollutes feedback, “tail chasing” can occur.
    - Crossover frequency rolloff is mandatory, e.g. in PFC circuits
  - presence of a Right-Half Plane Zero (RHPZ): you cannot cross over beyond 30% of the lowest RHPZ position
  - output undershoot specification:
    - Select crossover frequency based on undershoot specs

Don’t push the crossover frequency too far!!
How to force crossover and phase margin?

- The converter we want to compensate exhibits a transfer function.
- This is the power stage open-loop transfer function noted $H(s)$.
- On this plot, a crossover frequency is identified, $f_c$.
- The designer reads the gain deficiency and the phase rotation at $f_c$.
- It can sometimes be a gain excess, in PFC stages for instance.
- A compensator transfer function $G(s)$ is inserted so that it:
  - provides gain/attenuation at the crossover frequency: $|H(f_c)G(f_c)| = 1$
  - boosts the phase at the crossover: $\arg H(f_c) + \arg G(f_c) = -360^\circ + \phi_m$.

What do we mean by “phase boost”?

- Control theory instructs to keep $T(s)$ away from the point -1,j0.
- At the frequency where $|T(s)| = 1$, $\arg T(s)$ should be less than $-360^\circ$.
- To generate phase margin, we need to improve $\arg T(s)$ at crossover.
- The compensator $G$ is tailored to provide phase correction at $f_c$.
- The amount of needed phase correction is called the phase boost.
How to force crossover and phase margin?

- Here, we want a 4-kHz crossover point and a 60° phase margin.
- Build $G(s)$ so that $|G(4kHz)| = +21$ dB and $\arg G(4kHz) = -125°$.

**Diagram:**

- $|H(s)| = -21$ dB
- $\arg H(s) = -175°$
- $|G(4kHz)| = +21$ dB
- $\arg G(4kHz) = -125°$

**Equations:**

- $\arg H(f_c) + \arg G(f_c) = -360° + \phi_m$
- $\arg G(f_c) = -360° + \phi_m - \arg H(f_c)$
- $\arg G(f_c) = -125°$
- $\arg T(f_c) = -175 - 125 = -300°$
- $\phi_m = 60°$

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Poles, zeros and RHPZ

- A loop gain can be put under the following form:

\[ H(s) = \frac{N(s)}{D(s)} \]

- solving for \( N(s) = 0 \), the roots are called the **zeros**
- solving for \( D(s) = 0 \), the roots are called the **poles**

\[ H(s) = \frac{(s + 5k)(s + 30k)}{s + 1k} \]

Numerator roots:
\[ s_1 = -5k \]
\[ s_2 = -30k \]
\[ s_n = -1k \]
Denominator root:
\[ s_c = 5k \]
\[ s_c = 30k \]
\[ s_c = 1k \]
\[ f_c = \frac{5k}{2\pi} = 796 \text{ Hz} \]
\[ f_c = \frac{30k}{2\pi} = 4.77 \text{ kHz} \]
\[ f_c = \frac{1k}{2\pi} = 159 \text{ Hz} \]

The pole

- A pole creates a phase lag of \(-45^\circ\) at its cutoff frequency

\[ V_{\text{in}}(s) \]
\[ V_{\text{out}}(s) \]
\[ \frac{1}{1 + \frac{s}{RC}} \]

We can write the equation in different forms:

\[ \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{1 + \frac{s}{s_a}} = \frac{1}{1 + \frac{s}{s_b}} \]

where \( s_a = s \) and \( s_b = \frac{1}{RC} \)
The Pole

- Its magnitude at the cutoff frequency is -3 dB
- Its asymptotic phase, when in the LHP, at $f = \infty$ is $-90^\circ$
- The pole “lags” the phase

\[
V_{in}(s) = \frac{1}{1 + sRC} = \frac{1}{1 + \frac{s}{\omega_0}}
\]

\[
20 \log_{10} \left| \frac{V_{out}(s)}{V_{in}(s)} \right| = 20 \log_{10} \left| \frac{\frac{s}{\omega_0}}{1 + \frac{s}{\omega_0}} \right| = 20 \log_{10} \frac{1}{\sqrt{2}} = -3 \text{ dB} \quad \text{At } f = f_{p1}
\]

\[
\arg \frac{V_{out}(s)}{V_{in}(s)} = \arg \left( 1 - \frac{s}{\omega_0} \right) = -\arg(1) = -\pi / 4 \quad \text{At } f = f_{p1}
\]

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\]

The Zero

- A zero boosts the phase by $+45^\circ$ at its cutoff frequency

The general form of a zero:

\[
G(s) = 1 + \frac{s}{\omega_0}
\]
The zero

- Its magnitude at the cutoff frequency is +3 dB
- Its asymptotic phase, when in LHP, at \( f = \infty \) is +90°
- The zero "boosts" the phase

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = 1 + \frac{s}{a_b}
\]

\[
20 \log_{10} \left| \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \right| = 20 \log_{10} \left| 1 + \frac{s}{s_c} \right| = 20 \log_{10} \sqrt{2} = +3 \text{ dB} \quad \text{At } f = f_z
\]

\[
\arg \frac{V_{\text{out}}(s_c)}{V_{\text{in}}(s_c)} = \arg \left( 1 + \frac{s}{s_c} \right) = \arctan(1) = +\frac{\pi}{4} \quad \text{At } f = f_z
\]

\[
\arg \frac{V_{\text{out}}(\infty)}{V_{\text{in}}(\infty)} = \arg \left( 1 + \frac{\infty}{s_p} \right) = \arctan(\infty) = +\frac{\pi}{2} \quad \text{At } f = \infty
\]
Poles and zeros at the origin

- The integration time constant changes the 0-dB crossover frequency

\[ G(s) = \frac{1}{sRC} = \frac{1}{s/\omega_{po}} \]

\[ \omega_{po} = \frac{1}{RC} \]

S = -1 is the origin pole

S = -j is the 0-dB crossover pole frequency

The Right-Half-Plane Zero

- In a CCM boost, \( I_{out} \) is delivered during the off time: \( I_{out} = I_o = I_L (1 - D) \)

- If \( D \) brutally increases, \( D' \) reduces and \( I_{out} \) drops!

- What matters is the inductor current slew-rate

\[ \frac{d(V_i)}{dt} \]
**The Right-Half-Plane Zero**

- If $I_L(t)$ can rapidly change, $I_{out}$ increases when $D$ goes up

![Graph showing $I_L(t)$ and $I_{out}(t)$ with $D(t)$ and $V_{out}(t)$ over time.]

- If $I_L(t)$ is limited because of a big $L$, $I_{out}$ drops when $D$ increases

![Graph showing $I_L(t)$ and $I_{out}(t)$ with $D(t)$ and $V_{out}(t)$ over time.]

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---
The Right-Half-Plane Zero

- To limit the effects of the RHPZ, limit the duty ratio slew-rate
- Chose a crossover frequency equal to 20-30% of RHPZ position
  - A simple RHPZ can be easily simulated:

\[ V_{out}(s) = V_{in}(s) - V_{in}(s) \frac{R_1}{sC_1} = V_{in}(s) \left( \frac{s}{\omega_0} \right) \]

The negative sign confirms for the RHPZ presence.

The Right-Half-Plane Zero

- With a RHPZ we have a boost in gain but a lag in phase!
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Combining poles and zeros

- We know that a pole lags the signal phase by -90°
- A zero boosts the signal phase by +90°
  - if we combine both in $G(s)$, we can control the phase from:
    - 0° if the pole and the zero are coincident
    - +90° if the pole and the zero are split far away from each other
  - up to 180° with double pole/zero pair!

![Diagram](image)

**Parameters**

- $k = 1$
- $F_z = 1000$
- $F_p = 1000 \times k$
Combining poles and zeros

- When the pole and zero are coincident ($k = 1$), no boost
- The farther they are, the greater the boost
- As the pole/zero split apart, $f$ at which the boost peaks, changes

![Graph showing the relationship between pole and zero positions and the boost magnitude](image)

The equation where a pole and a zero are combined is:

$$G(s) = \left(\frac{1 + \frac{s}{s_z}}{1 + \frac{s}{s_p}}\right) = \frac{N}{D}$$

- The argument of a quotient is: $\arg N - \arg D$

$$\arg G(f) = \arctan\left(\frac{f}{f_z}\right) - \arctan\left(\frac{f}{f_p}\right)$$

- Where does the phase peak (the boost) occur?

$$\frac{d}{df} \left( \arctan\left(\frac{f}{f_z}\right) - \arctan\left(\frac{f}{f_p}\right) \right) = 0$$

Max boost occurs at:

$$f = \sqrt{\frac{f_z}{f_p}}$$
Do not forget the op amp

- In reality, poles and zeros are combined with an op amp
- To reduce the static error, we need a high dc gain
  - A pole at the origin is almost always part of \( G(s) \rightarrow \) integrator
  - An origin pole permanently lags the phase by \(-90^\circ\)
  - With the op amp, the minimum phase lag is: \(-90 - 180 = -270^\circ\)

SPICE shows a \(+90^\circ\) phase rotation rather than a \(-270^\circ\) value. Why?

Because of the modulo \(2\pi\) representation:

\[
\theta = \frac{3\pi}{2} \pm k \frac{2\pi}{k = 1}
\]

How to calculate the necessary boost?

- We know that \( \arg G(s) \) lags by \(-270^\circ\) for \( s = 0\)
- The arguments sum of \( G(f_c) \) and \( H(f_c) \) must stay away from \(-360^\circ\)
  - \( \varphi_m \) is the distance between \( [\arg G(f_c) + \arg H(f_c)] \) and \(-360^\circ\)

\[
\arg H(f_c) = -270^\circ + \text{BOOST} - \varphi_m = -360^\circ
\]

\[
\text{BOOST} = \varphi_m - \arg H(f_c) - 90^\circ
\]

- Assume a 4-kHz crossover frequency is wanted
  - \( \arg H(4k) = -68^\circ \); how much boost for a 70° phase margin?

\[
\text{BOOST} = 70 + 68 - 90 = 48^\circ \quad \Rightarrow \quad \arg G(4k) = -270 + 48 = -222^\circ
\]

- Combining the previous equations, we have:

\[
f_p = \left[ \tan(\text{boost}) + \sqrt{\tan^2(\text{boost}) + 1} \right] f_c = 2.6 \times 4k = 10.4 \text{ kHz}
\]

\[
f_c = \frac{\left( \frac{16k}{10.4k} \right)^2}{f_p} = 1.54 \text{ kHz}
\]
How to calculate the necessary boost?

- The pole has been placed at 10.4 kHz and the zero at 1.5 kHz

How to crossover at $f_c$ then?

- We know how to create the boost by placing 1 pole and 1 zero
- How do we now create the right gain at crossover?
- The final formula for $G(s)$ must include the 0-dB crossover pole:

$$G(s) = \frac{1 + \frac{s}{s_0}}{\frac{s}{s_0} + \frac{1 + \frac{s}{s_0}}{1 + \frac{s}{s_0}}} = \frac{s}{s_0} \left( 1 + \frac{s}{s_0} \right)$$

- By adjusting the 0-dB crossover pole frequency $f_{po}$, you can tailor the gain at crossover.
Shift $s_{po}$ to adjust the crossover gain

- The zero is fixed to get the proper phase boost.
- By adjusting the 0-dB crossover pole position, you adjust the gain at $f_c$.

This so-called mid-band gain makes $T(s)$ crossover at $f_c$.

Always write compensator transfer function with $G_0$:

$$G(s) = G_0 A(s)$$

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What is a type 1 amplifier?

- In some cases, you do not need phase boost at all
  - If \( \arg H(f_c) < -45^\circ \) within the band of interest:
    \[
    \arg H(f_c) - 270^\circ \leq -315^\circ \quad \quad \Rightarrow \quad \varphi_m \geq 45^\circ
    \]
  - A 45° phase margin is guaranteed

- There is an origin pole at \( s = 0 \)
  \[
  G(s) = -\frac{1}{s} = \frac{1}{s_{po}} \quad \quad \Rightarrow \quad \quad G(j\omega) = j\omega \frac{1}{s_{po}} \quad \quad \Rightarrow \quad \quad |G(f)| = \frac{f_{po}}{f}
  \]

- \( \arg G(f) = \arg (-1) - \arg \left( \frac{s}{s_{po}} \right) = -\pi - \arctan(\infty) = -\frac{3\pi}{2} \)

- Select \( f_{po} \) depending on the wanted gain at crossover:
  \[
  |G(1kHz)| = 20 \text{ dB} \quad \Rightarrow \quad |G(1kHz)| = 10^{\frac{20}{20}} = 10^1 = 10
  \]
  \[
  f_{po} = 10f_c = 10 \text{ kHz}
  \]

What Bode plot for the type 1?

- The type 1 does not provide phase boost at all

![Bode plot for type 1 amplifier](image)
What is a type 2 amplifier?

- In the vast majority of cases, phase boost is needed
- If the needed phase boost is less than 90°, a type 2 can do the job
- An origin pole plus a zero and a pole:

\[ G(s) = -\frac{\left(1 + \frac{s}{\omega}_o \right)}{\left(1 + \frac{s}{\omega}_p \right)} = -G_0 \left(1 + \frac{s}{\omega}_z \right) \]  
with \( G_0 = \frac{s}{\omega}_o \)

- The magnitude is derived as:
- The argument is found to be:

\[ |G(f)| = G_0 \sqrt{1 + \left(\frac{f}{f_p}\right)^2} \]  

\[ \arg G(f) = \arctan \left(\frac{f}{f_p}\right) - \pi - \arctan \left(\frac{f}{f_p}\right) \]

- The 0-dB crossover pole frequency is placed at: \( f_p = \omega_o f_c \)

Where to place the poles and zero?

- First, place the pole and zero for the needed phase boost
- Then adjust the origin pole 0-dB frequency at the right value
- 5-kHz crossover gain deficiency is -18 dB, required boost is +68°

\[ f_p = \left[ \tan (\text{boost}) + \sqrt{\tan^2 (\text{boost}) + 1} \right] f_c = 5.14 \times 5k = 25.7 \ kHz \]

\[ f_z = \frac{f_p^2}{25k} = 970 \ Hz \]

- A +18-dB gain is necessary at 5 kHz:

\[ |G(5kHz)| = 18 \ dB \quad \rightarrow \quad |G(5kHz)| = 10^{18/20} = 8 \]

\[ f_m = 8 f_z = 7.8 \ kHz \]
What Bode plot for a type 2?

- The type 2 provides phase boost up to +90°

\[ G(s) \]

\[ \arg G(s) = \arg G(s) = 158° \text{ or } -202° \]

\[ 90° \text{ or } 270° \]

\[ \text{boost} = 68° \]

What is a type 3 amplifier?

- Sometimes, a phase boost greater than 90° is needed
- By doubling the pole and zero, we can boost up to 180°

\[ G(s) = \frac{s}{s + s_o} \left( \frac{s + s_o}{s + s_p} \right) \]

\[ \arg G(f) = \arg N - \arg D \]

\[ \left| G(f) \right| = \frac{f_p}{f_o} \sqrt{\frac{1 + \left( \frac{f}{f_p} \right)^2}{1 + \left( \frac{f}{f_p} \right)^2}} \]

\[ \arg N = \arctan \left( -\frac{f}{f_p} \right) - \pi + \arctan \left( \frac{f}{f_p} \right) \]

\[ \arg D = \arctan \left( \frac{f}{f_p} \right) + \arctan \left( \frac{f}{f_p} \right) \]
Where to place the poles and zeros?

- Poles and zeros can be coincident (k factor) or split
- Place the double pole and the double zero to get the boost
- Then adjust the origin pole 0-dB frequency at the right value
- 5-kHz crossover gain deficiency is +10 dB, required boost is +158°

- If we consider coincident poles and zeros:

\[
Boost = 2 \left[ \arctan \left( \frac{f_p}{f_z} \right) - \arctan \left( \frac{f_p}{f_z} \right) \right] \quad f_c = 5 \text{ kHz} = \sqrt{f_p f_z}
\]

\[
f_{p,z} = \frac{f_c}{\tan \left( \frac{Boost}{4} \right)} = \frac{5k}{96.3m} = 52 \text{ kHz}
\]

- A +10-dB gain is necessary at 5 kHz:

\[
|G(5\text{ kHz})| = 10 \text{ dB} \quad \rightarrow |G(5\text{ kHz})| = 10^{20} = 3.2
\]

\[
f_{pc} = \frac{G_p}{f_c} \left( \frac{f_{z,2}}{f_{z,1}} \right)^2 = 147 \text{ Hz}
\]

What Bode plot for a type 3?

- The type 3 provides phase boost up to +180°

\[
|G(5\text{ kHz})| = +10 \text{ dB}
\]

\[
\arg G(s) - \arg G(s) = 248^\circ \text{ or } -112^\circ
\]

\[
\text{boost} = 158^\circ
\]
When to use these compensators?

- **Type 1** is used where no phase boost is necessary at crossover
  - If a 45° $\phi_m$ is ok, a type 1 can be used where arg $H(f_c) < 45^\circ$
  - Power Factor Correction circuits
  - Current mode power supplies in CCM, DCM or CrM (BCM)
  - Voltage-mode power supplies in DCM
    - Pure integrator, brings output overshoot

- **Type 2** is targeting applications where a phase boost is necessary
  - In the above examples where a $\phi_m$ larger than 45°is requested
    - Most popular choice for current mode converters

- **Type 3** is selected where a large phase boost is mandatory
  - This is the case for CCM voltage-mode converters
  - Generally, 2nd order and beyond types of transfer functions

How to implement these compensators?

- **Operational Amplifier**: most documented architecture
  - Virtual ground arrangement excludes the resistive divider ratio
  - High open-loop gain for reduced static error
  - Best flexibility for poles/zeros arrangement

- **Transconductance Amplifier**: mainly used in PFC circuits
  - Offers a means to sense the output voltage on the feedback pin
    - Less flexibility for type 3 arrangement
    - Transconductance value appears in the poles/zeros equations

- **TL431**: the most popular architecture
  - Combines an op amp and a reference voltage: cheapest approach
  - Easy interface with an optocoupler
    - Low open-loop gain
    - Biasing requirements hamper its flexibility
**Agenda**

- Feedback generalities
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- Boosting the phase at crossover
- Compensator types
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  - Practical implementations: the OTA
  - Practical implementations: the TL431
- Design examples
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**Designing the divider network**

- The design starts with the divider network ratio
- Make sure enough current circulates in the bridge:
  - it improves noise immunity
  - it shields you against offset current in the op amp
  - it degrades the no-load consumption...

\[
R_{\text{lower}} = \frac{2.5}{I_{\text{bridge}}}
\]

\[
R_{\text{upper}} = \frac{V_{\text{ref}} - 2.5}{I_{\text{bridge}}}
\]

\[
V_{\text{out}} = V_{\text{ref}} - 2.5 R_{\text{bridge}} - I_{\text{bias}} R_{\text{upper}}
\]

\[
I_{\text{bridge}} = \frac{(V_{\text{out}} - V_{\text{ref}})/R_{\text{bridge}}}{I_{\text{bias}}}
\]

\[
V_{\text{out}} = V_{\text{ref}} \left( \frac{R}{R_{\text{upper}}} + 1 \right) + R_{\text{lower}} I_{\text{bias}}
\]

If \( I_{\text{bias}} \ll I_{\text{bridge}} \):

\[
V_{\text{out}} \approx V_{\text{ref}} \left( \frac{R}{R_{\text{upper}}} + 1 \right)
\]

If \( I_{\text{bridge}} = 250 \, \mu\text{A} \rightarrow R_{\text{lower}} = 2.5/250 \mu\text{A} = 10 \, \text{k}\Omega
\]

*Watch out with PFC!*
Type 1 with an op amp

- Type 1 is an inverting integrator providing one pole at the origin

\[ G(s) = \frac{Z_f}{Z_i} = \frac{1}{sC R} = \frac{1}{s} \]

\[ \omega_{po} = \frac{1}{RC} \]

\[ G(s) = -\frac{1}{\omega_{po} s} \]

\[ |G(j\omega)| = \left| \frac{\omega_{po}}{\omega} \right| = \sqrt{\left(\frac{\omega_{po}}{\omega}\right)^2} = \omega_{po} \]

If you need a +21-dB gain to crossover at 4 kHz,
where to place the 0-dB crossover pole?

\[ f_{po} = G_f \cdot f_c = 10^{21/20} \times 4k = 44.8 \text{ kHz} \]
**Type 2 with an op amp (full analysis)**

- Type 2 keeps the origin pole but adds one zero and one extra pole

\[ \frac{Z_f}{Z_i} = \left(\frac{1}{sC_1} + R_2\right) \frac{1}{sC_2} \left(\frac{1}{sC_1} + R_2\right) + \frac{1}{sC_2} \]

Re-arrange

\[ G(s) = \frac{-R_2}{sR_i(C_1 + C_2)} \left(\frac{1}{sR_1} + \frac{1}{1+sR_1 \left[ \frac{C_iC_s}{C_1+C_2} \right]} \right) = \frac{-R_2}{1+sR_1 \left[ \frac{C_iC_s}{C_1+C_2} \right]} \]

Factor \( sR_iC_1 \)

In the gain expression, we have:

- \( G_0 = \frac{R_2}{R_1} \frac{C_1}{C_1 + C_2} \)
- \( \omega_z = \frac{1}{R_1C_1} \)
- \( \omega_p = \frac{1}{R_1 \frac{C_iC_s}{C_1+C_2}} \)

As \( \omega_z, \omega_p, G_0 \) and \( R_1 \) are given (boost, \( V_{out} \) etc.) how to get \( R_2 \)?

\[ G(f_z) = G_0 \frac{1 + \left( \frac{f_z}{f_p} \right)^2}{1 + \left( \frac{f_z}{f_p} \right)^2} \]

\[ R_2 = \frac{G_0R_f}{f_p - f_z} \left( \frac{f_z}{f_p} \right)^2 + 1 \]

Other component values are then extracted:

\[ C_1 = \frac{1}{2\pi R_2f_z} \]
\[ C_2 = \frac{C_1}{2\pi f_z C_1 R_2 - 1} \]
Type 2 with an op amp (simplified analysis)

- In most cases, $C_2$ is much smaller than $C_1$. Therefore:

$$G(s) = \frac{R_1}{R_2} \frac{1/sR_1C_1+1}{(1+sR_2C_2)} = -G_0 = \frac{1+s/s_p}{1+s/s_p}$$

$$G_0 = \frac{R_2}{R_1} \quad \omega_c = \frac{1}{R_2C_2} \quad \omega_p = \frac{1}{R_1C_1}$$

$$|G(f_c)| = \frac{R_2}{R_1} \frac{\left(1 + \left(\frac{f_c}{f_p}\right)^2\right)^{\frac{1}{2}}}{\left(1 + \left(\frac{f_c}{f_p}\right)^2\right)^{\frac{1}{2}}} \quad R_2 = G_0R_1 \left(\frac{f_c}{f_p}\right)^{2} + 1$$

Type 2 with an op amp – design example

- You need to provide a 15-dB gain at 5 kHz with a 50° boost.
- How to calculate the component values?

$$f_p = \left[\tan(\text{boost}) + \sqrt{\tan^2(\text{boost}) + 1}\right] f_c = 2.74 \times 5k = 13.7 \text{ kHz}$$

$$f_c = \frac{f_p^2}{f_p^2 - f_c^2} = \frac{25k}{13.7k} = 1.8 \text{ kHz}$$

$$R_2 = \frac{G_1R_1f_p}{f_p - f_c} \left(\frac{f_c^2}{f_p^2}\right)^{1/2} = 64.8 \text{ k\Omega}$$

$$C_1 = \frac{1}{2\pi R_1 f_c} = 1.3 \text{ nF} \quad C_2 = \frac{C_1}{2\pi f_c R_2} - 1 = 206 \text{ pF}$$

- You can use the simplified formula in the general case.
- For PFCs, $C_2$ is not small compared to $C_1$, use full formulas.
Type 2 with an op amp – Bode plot

-60.0 dB
-30.0 dB
0 dB
+30.0 dB
+60.0 dB

-270°
-230°
-190°
-150°
-110°

-270°
boost = 50°

10
100
1k
10k
100k

Type 2 with an op amp – start-up issue

- For low bandwidth systems, capacitor values can be large
- For a PFC circuit, crossover can be as low as 20 Hz
- For a zero at 10 Hz and a pole at 40 Hz, we have:

- At power up, \( V_{\text{out}} = 0 \) and \( R_i \gg R_{\text{lower}} \)
- \( V_{\text{err}} \) should go to the op amp \( V_{cc} \)
- Big caps are in the compensation path
- \( I_i \) is limited by \( R_i \) and \( R_{\text{lower}} \)
  - op amp output slowly rises
  - delays the full power delivery

\[
I_2 = I_i - I_3 = \frac{V_{\text{ref}}}{R_{\text{lower}}} - \frac{V_{\text{ref}} - V_{\text{out}}}{R_i}
\]

\[
I_1(t) = \frac{V_{\text{ref}}}{R_{\text{upper}}} + \frac{V_{\text{ref}} - V_{\text{out}}(t)}{R_i} - \frac{V_{\text{ref}}}{R_{\text{lower}}}
\]

\( V_{\text{out}} = 0 \), \( R_{\text{upper}} \gg R_{\text{lower}} \)
Type 2 with an op amp – start-up issue

- The capacitor charging acts as an inexpensive soft-start
- If too small, $V_{out}$ rises too quickly and an overshoot appears

\[
\begin{align*}
V(t) &= 1.2 \text{ ms} \\
I(t) &= 500 \mu\text{A}
\end{align*}
\]

Type 2 with op amp – a different arrangement

- In the compensator, all the current flows in $C_2$, the smallest value
- Why not placing it differently then?

\[
\begin{align*}
G(s) &= \frac{R_f + R_1}{R_1 C_1} \\
C_2 &= \frac{1}{\omega_i (R_1 + R_2)} \\
C_1 &= \frac{1}{G \omega_i} \\
R_2 &= \frac{R_i \omega_i}{\omega_p - \omega_i}
\end{align*}
\]
A simple macro can be written to calculate all the elements around the compensator for both options.

Type 2, Bode plot for both solutions

- Both curves perfectly superimpose on each other
Type 2 in a PFC circuit

- An average model is used to test both structures
- Start-up and transient response is studied in each case

![Circuit Diagram]

Replaced by the second type (opt 2)

Type 2 in a PFC circuit – transient response

- The small-signal response is similar ($f_c = 20$ Hz, $\phi_m = 60^\circ$)
- Overshoot is reduced by 5% in the second option

![Graphs of Transient Response]

Start-up sequence

Transient response
Type 3 with an op amp (full analysis)

- Type 3 keeps the origin pole but add a zero/pole pair

\[ Z_f = \left( \frac{1}{sC_1} + R_f \right) \sqrt{\frac{1}{sC_1} + R_1} + \frac{1}{sC_2} \]

\[ Z_f = \left( \frac{1}{sC_3} + R_f \right) R_2 \sqrt{\frac{1}{sC_3} + R_f} + R_3 \]

\[ V_{in} \]

\[ G(s) = \frac{1}{sC_1} \left( \frac{1 + \frac{s}{s_1}}{1 + \frac{s}{s_2}} \right) \left( \frac{1 + \frac{s}{s_3}}{1 + \frac{s}{s_4}} \right) \]

\[ G_0 = \frac{R_2}{R_1 C_1 + C_2} \quad \omega_1 = \frac{1}{R_2 C_1} \quad \omega_2 = \frac{1}{sC_2} \]

\[ \omega_{12}, \omega_{p12}, G_0 \text{ and } R_1 \text{ are given (boost, } V_{out} \text{ etc.) how to get } R_2? \]

\[ G(f_r) = G_0 \sqrt{\frac{1 + \left( \frac{f_r}{f_{p1}} \right)^2}{1 + \left( \frac{f_r}{f_{p2}} \right)^2} \frac{1 + \left( \frac{f_r}{f_{p3}} \right)^2}{1 + \left( \frac{f_r}{f_{p4}} \right)^2}} \]

\[ R_2 = \frac{G_0 R_1 f_{p1}}{f_{p1} - f_{p2}} \sqrt{\frac{1 + \left( \frac{f_r}{f_{p1}} \right)^2}{1 + \left( \frac{f_r}{f_{p3}} \right)^2} \frac{1 + \left( \frac{f_r}{f_{p1}} \right)^2}{1 + \left( \frac{f_r}{f_{p4}} \right)^2}} \]
Type 3 with an op amp (simplified analysis)

- Extract the rest of the elements:
  
  \[
  C_1 = \frac{1}{2\pi f_p R_2} \quad C_2 = \frac{C_1}{2\pi f_p C_i R_2 - 1} \quad C_3 = \frac{f_{p_2} - f_{p_3}}{2\pi f_{p_{upper}} f_p f_{p_2}} \quad R_3 = \frac{R_2 f_{p_2}}{f_{p_3} - f_{p_2}}
  \]

- In most cases, \( C_2 \ll C_1 \) and \( R_3 \ll R_1 \). Therefore:

  \[
  G(s) = \frac{R_2 s R_i C_i + 1}{R_i (1 + s R_i C_i) s C_i R_i + 1} \quad C_1 = \frac{1}{2\pi f_p R_2} \quad C_2 = \frac{1}{2\pi f_p R_2} \quad C_3 = \frac{1}{2\pi f_p R_3} \quad R_2 = \frac{R_2 f_{p_2}}{f_{p_3} - f_{p_2}}
  \]

\[
\left| G(f) \right| = \frac{R_2}{R_1} \left[ \frac{1 + \left( \frac{f_{p_3}}{f_p} \right)^2}{1 + \left( \frac{f_{p_2}}{f_p} \right)^2} \right] \left[ \frac{1 + \left( \frac{f_{p_3}}{f_{p_3}} \right)^2}{1 + \left( \frac{f_{p_3}}{f_{p_3}} \right)^2} \right]
\]

Type 3 with an op amp – design example

- You need to provide a -10-dB gain at 5 kHz with a 145° boost

\[
\begin{align*}
  f_{p_{upper}} &= \frac{f_p}{\tan(45 - \frac{\text{Boost}}{4})} = \frac{5k}{154m} = 32.5\,\text{kHz} \\
  f_{p_3} &= \frac{f_p}{f_{p_{upper}}} = \frac{25k}{32.5k} = 769\,\text{Hz}
\end{align*}
\]

\[
R_2 = \frac{G_s R f_{p_{upper}}}{f_{p_3} - f_p} \left[ \frac{1 + \left( \frac{f_{p_3}}{f_{p_3}} \right)^2}{1 + \left( \frac{f_{p_3}}{f_{p_3}} \right)^2} \right] = 498\,\Omega
\]

\[
C_1 = 415\,\text{pF} \quad C_2 = 10\,\text{nF} \quad C_3 = 20\,\text{nF} \quad R_3 = 242\,\Omega
\]
Type 3 with an op amp – Bode plot

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Type 1 with an OTA

- A type 1 with an OTA involves the transconductance value $g_m$

$$G(s) = \frac{1}{sR_{\text{linear}} + R_i}$$

$$G(s) = \frac{1}{sR_{\text{upper}} + R_{\text{lower}}}$$

$$G(s) = \frac{1}{sR_{\text{upper}} + R_{\text{lower}} + R_i}$$

$$G(s) = \frac{1}{sR_{\text{upper}} + R_{\text{lower}} + R_i}$$

$$G(s) = \frac{1}{sR_{\text{upper}} + R_{\text{lower}} + R_i}$$

$$G(s) = \frac{1}{sR_{\text{upper}} + R_{\text{lower}} + R_i}$$

The divider network now enters the picture!

Type 1 with an OTA – design example

- A Borderline PFC with a MC33262 controller: open-loop gain test
Type 1 with an OTA – design example

A type 1 as exemplified in the data-sheet gives a weak $\phi_m$!

$$|H(f)|\quad G_{fe} = 38\, \text{dB}$$

$$\arg H(f)\quad \phi = -71^\circ$$

G_{fe} = 12.6 m\quad f_{po} = 12.6 m \times 7 = 88\, \text{mHz}\quad \rightarrow\quad C_1 = 560\, \text{nF}$$

Type 2 with an OTA

A type 2 with an OTA requires the addition of a resistor and a cap.

$$G(s) = \frac{R_{\text{lower}} \text{gm}}{R_1 + R_{\text{lower}}} \left( \frac{1}{s C_1} + \frac{1}{s C_2} \right)$$

$$G(s) = -\frac{R_1 C_1 \text{gmR}_{\text{lower}}}{C_1 + C_2 + R_{\text{lower}}} \frac{\sqrt{1 + \left( f / f_p \right)^2}}{1 + \left( f / f_p \right)^2}$$

$$\omega_z = \frac{1}{R_2 C_1}\quad \omega_p = \frac{1}{R_f \left( \frac{C_1 C_2}{C_1 + C_2} \right)}\quad R_2 = \frac{G_{fz} f_p}{f_p - f_z} \frac{R_{\text{lower}} + R_1 \sqrt{1 + \left( f / f_p \right)^2}}{1 + \left( f / f_p \right)^2}$$
Type 2 with an OTA – design example

- You need to provide a 15-dB gain at 5 kHz with a 50° boost.
- The poles and zero position are that of the op amp design.

\[ R_2 = 260 \, k\Omega \quad C_1 = 340 \, pF \quad C_2 = 52 \, pF \quad g_m = 50 \, \mu S \quad R_1 = R_{\text{shv2}} = 10 \, k\Omega \]

PFC response: OTA versus op amp

- The poles/zero are placed at the same location as in the op amp case.

\[ V_{\text{peak}} = 438 \, V \quad V_{\text{peak}} = 430 \, V \]
Type 3 with an OTA

A type 3 with an OTA lets you boost the phase up to 180°. In theory...

\[ G(s) = \frac{\frac{R_{\text{linear}} \cdot \text{gm}}{R_1 + R_{\text{linear}}}} {sC_1 (R_1 + R_1) + 1 + \frac{1}{sR_1 C_1}} \]

If \( C_2 \ll C_1 \)

\[ G(s) = \frac{\frac{R_{\text{linear}} \cdot \text{gm} R_2}{R_{\text{linear}} + R_1}} {sC_3 \left( \frac{R_{\text{linear}}}{R_{\text{linear}} + R_1} \right) + 1 + \frac{1}{sR_2 C_2}} \]

The extraction of the component values leads to complicated equations:

- First calculate \( R_3 \) but \( R_{\text{lower}} \) plays a role:
  - No virtual ground as with the op amp!

\[ R_3 = \frac{f_z V_{\text{out}} - f_p V_{\text{ref}}}{V_{\text{ref}} V_{\text{out}} \left( f_p - f_z \right)} R_{\text{linear}} \left( V_{\text{out}} - V_{\text{ref}} \right) \]

- Then calculate \( R_2 \) to crossover at the right frequency:

\[ R_2 = \frac{\sqrt{\left( f_{p1}^2 + f_{p2}^2 \right) \left( f_{z1}^2 + f_{z2}^2 \right) \left( f_{p1}^2 + f_{z1}^2 \right) \left( f_{p2}^2 + f_{z2}^2 \right)}} {f_{z1} f_{z2} + f_{p1}^2 + f_{p2}^2 + f_{p1} f_{p2}^2} \]

\[ C_1 = \frac{1}{2\pi f_z} \]

\[ C_2 = \frac{1}{2\pi f_p} \]

\[ C_1 = \frac{1}{f_z (R_1 + R_1)} \]
Type 3 with an OTA

- If we look at $R_3$ definition, its numerator can be null:
  \[
  f_{z_2} \frac{V_{out}}{p_2} - f_{p_2} V_{ref} = 0
  \]
  \[
  f_{z_2} > \frac{V_{ref}}{V_{out}} f_{p_2}
  \]

  
  
  If $V_{out} = 12$ V and $V_{ref} = 2.5$ V
  
  \[
  f_{z_2} > \frac{f_{p_2}}{4}
  \]
  \[
  f_{z_2} > 10 \text{ kHz}
  \]

  
  
  If $V_{out} = 400$ V and $V_{ref} = 2.5$ V
  
  \[
  f_{z_2} > \frac{f_{p_2}}{400}
  \]
  \[
  f_{z_2} > 63 \text{ Hz}
  \]

  
  
  Less freedom to place the second pole and zero: limited boost!

Type 3 with an OTA – a design example

Type 3 – OTA
Type 3 with an OTA – a design example

- The op amp and the OTA designs perfectly match!

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The TL431 programmable zener

- The TL431 is the most popular choice in nowadays designs
- It associates an open-collector op amp and a reference voltage
- The internal circuitry is self-supplied from the cathode current
- When the R node exceeds 2.5 V, it sinks current from its cathode

The TL431 is a shunt regulator

The TL431 programmable zener

- The TL431 lends itself very well to optocoupler control

- $R_{LED}$ connected to $V_{out}$ offers a direct path to the LED: fast lane!
The TL431 programmable zener

- At high frequencies, the TL431 ac output is zero, $C_i$ is a short-circuit
- $R_{LED}$ alone fixes the fast lane gain

$$V_{out}(s)$$

$$R_{LED}$$

$$V_{FB}(s)$$

$$I_1 = \frac{V_{out}(s)}{R_{LED}}$$

$$V_{FB}(s) = -CTR \cdot R_{pullup} \cdot I_1$$

$$I_1 = \frac{V_{out}(s)}{R_{LED}}$$

$$\frac{V_{FB}(s)}{V_{out}(s)} = -CTR \frac{R_{pullup}}{R_{LED}}$$

- $R_{LED}$ must also leave enough headroom to the TL431: upper limit!

The TL431 programmable zener

- $R_{LED}$ cannot exceed a certain value because of bias limits
- $V_{FB}$ must swing between $V_{CE,sat}$ and $V_{cc}$

$$I_{C,max} = \frac{V_{ce} - V_{CE,sat}}{R_{pullup}}$$

$$I_{C,max} = \frac{V_{dd} - V_{CE,sat}}{R_{pullup} \cdot CTR_{min}} + I_{bias}$$

$$I_{LED} = 1V$$

$$V_{bias} = \frac{1V}{R_{bias}}$$

$$V_{out} - V_{j} = \frac{V_{out} - V_{TL431,min}}{R_{LED}}$$

$$V_{out} = 2.5V$$

$$R_{LED_{max}} \leq \frac{V_{out} - V_{j} - V_{TL431,min}}{V_{dd} - V_{CE,sat} + I_{bias} \cdot CTR_{min} \cdot R_{pullup}} \cdot R_{pullup} \cdot CTR_{min}$$

- dc representation
The TL431 – the static gain limit

Let us assume the following design:

\[
R_{LED,\text{max}} \leq \frac{5 - 1 - 2.5}{4.8 - 0.3 + 1 m \times 0.3 \times 20 k} \times 20 k \times 0.3
\]

\[
R_{LED,\text{max}} \leq 857 \Omega
\]

\[
G_o > CTR \frac{R_{\text{pullup}}}{R_{LED}} > 0.3 \frac{20}{0.857} > 7 \text{ or } 17 \text{ dB}
\]

In designs where \( R_{LED} \) fixes the gain, \( G_o \) cannot be below 17 dB

You cannot “amplify” by less than 17 dB

The TL431 – the static gain limit

You must identify the areas where compensation is possible

\( f_c > 500 \text{ Hz} \)

Requires 17 dB or more

Requires less than 17 dB of gain

\( f_c > 500 \text{ Hz} \)
**TL431 – injecting bias current**

- Make sure enough current always biases the TL431: $I_{bias} > 1$ mA
- If not, its open-loop suffers – a 10-dB difference can be observed!

**TL431 – small-signal analysis**

- The TL431 is an open-collector op amp with a reference voltage
- Neglecting the LED dynamic resistance, we have:

$$V_{out}(s) = \frac{V_{out}(s) - V_{op}(s)}{R_{LED}}$$

$$V_{op}(s) = -V_{out}(s) \frac{sC_1}{R_i} = -V_{out}(s) \frac{1}{sR_iC_1}$$

$$I_i(s) = V_{out}(s) \frac{1}{R_{LED} + \frac{1}{sR_iC_1}}$$

We know that: $V_{FB}(s) = -CTR \cdot R_{pullup} \cdot I_i$

$$V_{FB}(s) = \frac{R_{pullup} \cdot CTR}{R_{LED}} \left[ \frac{1 + sR_iC_1}{sR_iC_1} \right]$$

$$V_{out}(s) = \frac{1}{R_{LED}} \left[ 1 + \frac{1}{sR_iC_1} \right]$$
The optocoupler also features a parasitic capacitor

\[ C_2 = C \parallel C_{\text{opto}} \]

It comes in parallel with \( C_2 \) and must be accounted for.
The optocoupler must be characterized to know where its pole is.

Adjust $V_{\text{bias}}$ to have $V_{\text{FB}}$ at 2-3 V to be in linear region, then ac sweep.

The pole in this example is found at 4 kHz.

Another design constraint!

The TL431 in a type 1 compensator

To make a type 1 (origin pole only) neutralize the zero and the pole.

Once neutralized, you are left with an integrator.

$G(s) = \frac{1}{\omega_p s}$
TL431 type 1 design example

We want a 5-dB gain at 5 kHz to stabilize the 5-V converter

\[ V_{\text{out}} = 5 \text{ V} \]
\[ V_{\text{f}} = 1 \text{ V} \]
\[ V_{\text{TL431,min}} = 2.5 \text{ V} \]
\[ V_{\text{in}} = 4.8 \text{ V} \]
\[ V_{\text{CE,\text{out}}} = 300 \text{ mV} \]
\[ I_{\text{bias}} = 1 \text{ mA} \]
\[ \text{CTR}_{\text{min}} = 0.3 \]
\[ R_{\text{pullup}} = 20 \text{ k}\Omega \]

\[ G_{\text{f}} = 10^{\pi f_{\text{c}}} = 1.77 \]
\[ f_{\text{c}} = 5 \text{ kHz} \]
\[ R_{i} = 10 k\Omega \]

\[ C_{\text{opto}} = 2 \text{ nF} \]
\[ C_{\text{pole}} = \frac{\text{CTR}}{2\pi f_{\text{c}} R_{\text{LED}}} \]
\[ C_{\text{pole}} = \frac{\text{CTR}_{\text{min}}}{2\pi f_{\text{c}} R_{\text{LED}}} \]

Apply 15% margin

\[ R_{\text{LED,\text{max}}} \leq 857 \Omega \]
\[ R_{\text{LED}} = 728 \Omega \]

\[ G = 10^{\pi f_{\text{c}}} \]
\[ f_{\text{c}} = 5 \text{ kHz} \]

\[ R_{\text{pullup}} = 20 k\Omega \]

\[ C_{\text{opto}} = \frac{R_{\text{pullup}}}{R_{\text{LED}}} \]

SPICE can simulate the design – automate elements calculations…
TL431 type 1 design example

- We have a type 1 but 1.3 dB of gain is missing?

\[ G(s) \]

3.7 dB

- \[ \arg G(s) \]

- The 1-kΩ resistor in parallel with the LED is an easy bias.
- However, as it appears in the loop, does it affect the gain?

\[ V_{FB} = I_e R_{pullup} = I_L R_{pullup} \text{CTR} \]

\[ I_L = I_1 \frac{R_{bias}}{R_{bias} + R_d} \]

\[ I_L = \frac{V_{out}}{R_{LED} + R_{bias} || R_d + R_{bias} + R_d} \]

\[ V_{FB} \mid_{s=0} = \frac{R_{pullup} \text{CTR}}{R_{LED} + R_{bias} || R_d + R_{bias} + R_d} \]

- Both bias and dynamic resistances have a role in the gain expression.
TL431 type 1 design example

- A low operating current increases the dynamic resistor $R_d$

$$R_{pullup} = 20 \, k\Omega, \, I_p = 300 \, \mu A \, (CTR = 0.3)$$

$$R_d = 158 \, \Omega$$

$$R_{pullup} = 1 \, k\Omega, \, I_p = 1 \, mA \, (CTR = 1)$$

$$R_d = 38 \, \Omega$$

- Make sure you have enough LED current to keep $R_d$ small

TL431 type 1 design example

- The pullup resistor is 1 kΩ and the target now reaches 5 dB

$$|G(x)|$$

$5 \, \text{dB}$

$$\arg G(x)$$

TL431
The TL431 in a type 2 compensator

- Our first equation was already a type 2 definition, we are all set!

\[ G_o = \text{CTR} \frac{R_{\text{pullup}}}{R_{\text{LED}}} \]

\[ \omega_p = \frac{1}{R_C \omega} \]

\[ \omega_z = \frac{1}{R_C \omega} \]

\[ \omega_p = \frac{1}{R_C \omega} \]

\[ \omega_z = \frac{1}{R_C \omega} \]

TL431 type 2 design example

- You need to provide a 15-dB gain at 5 kHz with a 50° boost
- The output voltage is 12 V
- The poles and zero position are that of the op amp design

\[ G_o = \text{CTR} \frac{R_{\text{pullup}}}{R_{\text{LED}}} = 10^{15/20} = 5.62 \quad f_p = 13.7 \text{ kHz} \quad f_z = 1.8 \text{ kHz} \]

- With a 250-µA bridge current, the divider resistor is made of:

\[ R_{\text{dev}} = 2.5/250\mu A = 10 \text{ k}\Omega \quad R_i = (12 - 2.5)/250\mu A = 38 \text{ k}\Omega \]

- The pole and zero respectively depend on \( R_{\text{pullup}} \) and \( R_1 \):

\[ C_2 = \frac{1}{2\pi f_p R_{\text{pullup}}} = 581 \text{ pF} \quad C_1 = \frac{1}{2\pi f_z R_i} = 2.3 \text{ nF} \]

- The LED resistor depends on the needed mid-band gain:

\[ R_{\text{LED}} = \frac{R_{\text{pullup}} \text{CTR}}{G_o} = 1.06 \text{ k}\Omega \quad \text{ok} \quad R_{\text{LED, max}} \leq 4.85 \text{ k}\Omega \]
TL431 type 2 design example

- The optocoupler is still at a 4-kHz frequency:
  \[ C_{pole} = \frac{2\,\text{nF}}{2\,\text{kHz}} \] Already above!

- Type 2 pole capacitor calculation requires a 581-pF cap.!

  The bandwidth cannot be reached, reduce \( f_c \)!

- For noise purposes, we want a minimum of 100 pF for \( C \)
- With a total capacitance of 2.1 nF, the highest pole can be:
  \[ f_pole = \frac{1}{2\pi R_{pullup} C} = \frac{1}{6.28 \times 20k \times 2.1n} = 3.8\,\text{kHz} \]

- For a 50° phase boost and a 3.8-kHz pole, the crossover must be:
  \[ f_c = \frac{f_p}{\tan(\text{boost}) + \sqrt{\tan^2(\text{boost}) + 1}} = 1.4\,\text{kHz} \]

TL431 type 2 design example

- The zero is then simply obtained:
  \[ f_z = f_p \frac{f_z^2}{f_p} = 516\,\text{Hz} \]

- We can re-derive the component values and check they are ok
  \[ C_z = \frac{1}{2\pi f_p R_{pullup}} = 2.1\,\text{nF} \quad C_i = \frac{1}{2\pi f_z R_i} = 8.1\,\text{nF} \]

- Given the 2-nF optocoupler capacitor, we just add 100 pF

- In this example, \( R_{LED,\text{max}} \) is 4.85 kΩ

  \[ G_0 > \text{CTR} \frac{R_{pullup}}{R_{LED}} > 0.3 \frac{20}{4.85} > 1.2 \text{ or } 1.8 \text{ dB} \]

- You cannot use this type 2 if an attenuation is required at \( f_c \)!
TL431 type 2 design example

- The 1-dB gain difference is linked to $R_d$ and the bias current.

TL431 – suppressing the fast lane

- The gain problem comes from the fast lane presence.
- Its connection to $V_{out}$ creates a parallel input.
- The solution is to hook the LED resistor to a fixed bias.
The equivalent schematic becomes an open-collector op amp.

The small-signal ac representation puts all sources to 0.
TL431 – suppressing the fast lane

- The op amp can now be wired in any configuration!
- Just keep in mind the optocoupler transmission chain

\[ O(s) = \frac{R_{\text{pullup}}}{R_{\text{LED}}} \frac{CTR}{1 + sR_{\text{pullup}}C_{\text{pole}}} \]

- Wire the op amp in type 2A version (no high frequency pole)

\[ G_i(s) = \frac{1 + R_2C_1}{sR_1C_1} \]

- When cascaded, you obtain a type 2 with an extra gain term

\[ G(s) = \frac{R_{\text{pullup}}C_{\text{CTR}}}{R_{\text{LED}}} \frac{1 + R_2C_1}{sR_1C_1 \left( 1 + sR_{\text{pullup}}C_{\text{pole}} \right)} \]

TL431 type 2 design example – no fast lane

- We still have a constraint on \( R_{\text{LED}} \) but only for dc bias purposes

\[ R_{\text{LED, max}} \leq \frac{V_j - V_f - V_{\text{TL431, min}}}{V_{\text{dd}} - V_{\text{CE, sat}} + I_{\text{bias}}C_{\text{CTR, min}}R_{\text{pullup}}R_{\text{CTR, min}}} \]

- You need to attenuate by -10-dB at 1.4 kHz with a 50° boost
- The poles and zero position are that of the previous design

\[ \begin{align*} V_j &= 6.2 \text{ V} \\ V_f &= 1 \text{ V} \\ V_{\text{TL431, min}} &= 2.5 \text{ V} \\ V_{\text{dd}} &= 4.8 \text{ V} \\ V_{\text{CE, sat}} &= 300 \text{ mV} \\ I_{\text{bias}} &= 1 \text{ mA} \\ C_{\text{CTR, min}} &= 0.3 \\ R_{\text{pullup}} &= 20 \text{ k}\Omega \\ R_{\text{LED, max}} &= 1.5 \text{ k}\Omega \quad \rightarrow \quad R_{\text{LED}} = 1.27 \text{ k}\Omega \end{align*} \]

- Apply 15% margin

\[ f_z = 516 \text{ Hz} \quad f_p = 3.8 \text{ kHz} \]
TL431 type 2 design example – no fast lane

- We need to account for the extra gain term:
  \[ G_2 = \frac{R_{\text{pullup}}}{R_{\text{LED}}} \times \frac{20k}{1.27k} = 0.3 \times 4.72 \]

- The required total mid-band attenuation at 1.4 kHz is -10 dB
  \[ G_f = 10^{-10/20} = 0.316 \]

- The mid-band gain from the type 2A is therefore:
  \[ G_1 = \frac{G_0}{G_2} = \frac{0.316}{4.72} = 0.067 \text{ or } -23.5 \text{ dB} \]

- Calculate \( R_2 \) for this attenuation:
  \[ R_2 = G_2 R_\text{upper} \left( \frac{V_f}{f_c} \right)^2 + 1 \]

- An automated simulation helps to test the calculation results.
TL431 type 2 design example – no fast lane

- The simulation results confirm the calculations are ok

![Graph showing dB vs frequency ranging from 10 to 100 kHz and -30 dB @ 1.4 kHz]

- arg $G(s)$

- $\text{arg } G(s)$

- $\text{arg } G(s)$

The TL431 in a type 3 compensator

- The type 3 with a TL431 is difficult to put in practice

- $f_n = \frac{1}{2\pi R_c}$

- $f_p = \frac{1}{2\pi (R_{\text{LED}} + R_c) C_p}$

- $f_n = \frac{1}{2\pi R_c}$

- $f_p = \frac{1}{2\pi R_{\text{pullup}} \left( C_2 \parallel C_{\text{opto}} \right)}$

- $G = \frac{R_{\text{pullup}}}{R_{\text{LED}}}$

- $R_{\text{LED}}$ fixes the gain and a zero position

- Suppress the fast lane for an easier implementation!
The TL431 in a type 3 compensator

- Once the fast lane is removed, you have a classical configuration

\[
\begin{align*}
V_{dd} & \quad R_{\text{popup}} \quad R_{\text{LED}} \\
V_{i} & \quad R_{1} \quad R_{2} \quad R_{3} \\
V_{out} & \quad C_{1} \quad C_{2} \quad C_{3}
\end{align*}
\]

\[
\begin{align*}
R_{\text{popup}} & = R_{\text{LED}} \\
R_{1} & = R_{2} \\
R_{3} & = R_{\text{bias}} \\
C_{1} & = C_{2} \\
C_{3} & = \text{bias}
\end{align*}
\]

\[
\begin{align*}
f_{p1} & = \frac{1}{2\pi R_{1} C_{1}} \\
f_{p2} & = \frac{1}{2\pi R_{2} C_{2}} \\
f_{z1} & = \frac{1}{2\pi R_{3} C_{3}} \\
f_{z2} & = \frac{1}{2\pi R_{\text{popup}} (C_{2} || C_{3})}
\end{align*}
\]

\[
G = \frac{R_{\text{popup}} \cdot \text{CTR}}{R_{\text{LED}}}
\]

TL431 type 3 design example – no fast lane

- We want to provide a 10-dB attenuation at 1 kHz
- The phase boost needs to be of 120°
- Place the double pole at 3.7 kHz and the double zero at 268 Hz
- Calculate the maximum LED resistor you can accept, apply margin

\[
R_{\text{LED, max}} \leq \frac{V_{i} - V_{f} - V_{\text{TL431, min}}}{V_{dd} - V_{CE, sat} + I_{\text{bush}} \cdot \text{CTR}_{\text{min}}} \cdot R_{\text{popup}} \cdot \text{CTR}_{\text{min}} \leq 1.5 \, k\Omega \times 0.85 \rightarrow 1.3 \, k\Omega
\]

- We need to account for the extra gain term:

\[
G_{2} = \frac{R_{\text{popup}} \cdot \text{CTR}}{R_{\text{LED}}} = \frac{20k}{1.3k} \cdot 0.3 = 4.6
\]

- The required total mid-band attenuation at 1 kHz is -10 dB

\[
G_{f} = 10^{-10/20} = 0.316
\]
TL431 type 3 design example – no fast lane

- The mid-band gain from the type 3 is therefore:

\[ G_i = \frac{G_0}{G_2} = \frac{0.316}{4.6} = 0.068 \text{ or } -23.3 \text{ dB} \]

- Calculate \( R_2 \) for this attenuation:

\[
R_2 = \frac{G_i R_1 f_p}{f_p - f_n} \left[ \frac{1 + \left( \frac{f_n}{f_p} \right)^2}{1 + \left( \frac{f_n}{f_p} \right)^2} \right] = 744 \, \Omega
\]

- \( C_1 = 800 \, nF \quad C_2 = 148 \, pF \quad C_3 = 14.5 \, nF \quad C_{opto} = 2 \, nF \)

- The optocoupler pole limits the upper double pole position
- The maximum boost therefore depends on the crossover frequency

---

TL431 type 3 design example – no fast lane

- The decoupling between \( V_{out} \) and \( V_{bias} \) affects the curves

[Graph showing decoupling between \( V_{out} \) and \( V_{bias} \)]
Pushing the opto pole with the cascode

- The optocoupler pole is clearly a limiting factor
- A possibility exists to push its position to a higher region
  - The cascode fixes the optocoupler collector potential
  - It neutralizes the Miller capacitance of the optocoupler

With cascode

\[ f_p = 4.5 \text{ kHz} \]

Testing the TL431 fast lane structures

- Simulations are a good indication, but lab. results are better
- The TL431 needs to be exactly biased at \( V_{\text{out}} \) to ac sweep it
- A simple low-bandwidth op amp can do the job!
Testing the TL431 fast lane structures

- The results confirm the calculation procedures: a type 2

- Boost = 50°
- Gain = 0 dB
- $f_c = 1$ kHz

Testing the TL431 fast lane structures

- The results confirm the calculation procedures: a type 3

- Boost = 116°
- Gain = 17 dB
- $f_c = 1$ kHz
Agenda

- Feedback generalities
- The divider and the virtual ground
- Phase margin and crossover
- Poles and zeros
- Boosting the phase at crossover
- Compensator types
- Practical implementations: the op amp
- Practical implementations: the OTA
- Practical implementations: the TL431

Design examples

- A real case study
- Conclusion

Design Example 1 – a single-stage PFC

- The single-stage PFC is often used in LED applications
- It combines isolation, current-regulation and power factor correction
- Here, a constant on-time BCM controller, the NCP1608, is used
Design Example 1 – a single-stage PFC

- Once the converter elements are known, ac-sweep the circuit
- Select a crossover low enough to reject the ripple, e.g. 20 Hz

Given the low phase lag, a type 1 can be chosen
- Use the type 2 with fast lane removal where \( f_p \) and \( f_c \) are coincident

\[
H(s) = -\frac{2.5 \text{ dB}}{20 \text{ Hz}}
\]

\[
\text{arg} H(s) = -11^\circ
\]

\[
G(s) = \frac{6.1 \text{ k}\Omega}{10 \text{ k}\Omega}
\]

\[
I_{\text{on}} \text{ generation}
\]

\[
\phi_m = 90^\circ
\]

\[
f_c = 19 \text{ Hz}
\]
Design Example 1 – a single-stage PFC
- A transient simulation helps to test the system stability

Design Example 2 – a 300-W PFC
- A CCM PFC is delivering 300 W in universal mains
- Use an average model to plot its transfer function
Design Example 2 – a 300-W PFC

- Select the bandwidth at high line: 20 Hz

- Use a type 2 configuration and boost by 50°
Design Example 2 – a 300-W PFC

- Test the transient response and see dynamic enhancer effects

Design example 3: a DCM flyback converter

- We want to stabilize a 20-W DCM adapter
- $V_{in} = 85$ to $265$ V rms, $V_{out} = 12$ V/1.7 A
- $F_{sw} = 65$ kHz, $R_{pullup} = 20$ kΩ
- Optocoupler is SFH-615A, pole is at 6 kHz
- Cross over target is 1 kHz
- Selected controller: NCP1216

1. Obtain a power stage open-loop Bode plot, $H(s)$
2. Look for gain and phase values at cross over
3. Compensate gain and build phase at crossover, $G(s)$
4. Run a loop gain analysis to check for margins, $T(s)$
5. Test transient responses in various conditions
Design example 3: a DCM flyback converter

- Capture a SPICE schematic with an averaged model

![SPICE schematic with averaged model](image)

- Look for the bias points values: $V_{out} = 12$ V, ok

Design example 3: a DCM flyback converter

- Observe the open-loop Bode plot and select $f_c$: 1 kHz

![Bode plot](image)
Design example 3: a DCM flyback converter

- Apply $k$ factor or other method, get $f_z$ and $f_p$
  - $f_z = 3.5 \text{ kHz}$ $f_p = 4.5 \text{ kHz}$

$$f_z = 3.5 \text{ kHz} \quad f_p = 4.5 \text{ kHz}$$

- $k$ factor gave $C = 3.8 \text{nF}$
- install $C_z = 3.8n - 1.3n = 2.5 \text{nF}$
  - $C_{opt} = 1.3 \text{nF}$

Design example 3: a DCM flyback converter

- Check loop gain and watch phase margin at $f_c$
  - $\phi_m = 60^\circ$
Design example 3: a DCM flyback converter

- Sweep ESR values and check margins again

![Graph showing waveforms](image)

Design example 4: a CCM forward converter

- We have designed a 5-V/20-A telecom input converter
- We use the NCP1252, fixed-frequency current-mode

- We need high dc gain, an op amp is adopted, $f_c = 10$ kHz
Design example 4: a CCM forward converter

- Despite the op amp, we still have a fast lane issue!

\[
I_{LED}(s) = \frac{V_{out}(s) - V_{spump}(s)}{R_{LED}} \quad \rightarrow \quad \frac{V_{FB}(s)}{V_{out}(s)} = -\frac{R_{pullup} \cdot CTR}{R_{LED}} \left[ \frac{1 + sR_{C1}}{sR_{C1}(1 + sR_{pullup}C_2)} \right]
\]

- The LED resistor still limits the gain you can get:

\[
R_{LED,\text{max}} \leq \frac{V_{out} - V_f - V_{spump,\text{min}}}{V_{dd} - V_{CE,\text{sat}}} \cdot R_{pullup} \cdot CTR_{\text{min}}
\]

\[
\begin{align*}
V_{out} &= 5 \text{ V} \\
V_f &= 1 \text{ V} \\
V_{spump,\text{min}} &= 150 \text{ mV} \\
V_{dd} &= 4.8 \text{ V} \\
V_{CE,\text{sat}} &= 300 \text{ mV} \\
CTR &= 0.5 \\
R_{pullup} &= 3.3 \text{ k}\Omega
\end{align*}
\]

\[
R_{LED,\text{max}} \leq 1.4 \text{ k}\Omega
\]

- In this case, we cannot provide less than:

\[
G_0 = \frac{R_{pullup} \cdot CTR}{R_{LED}} = \frac{3.3 \text{ k} \times 0.5}{1.4} = 1.17 \text{ dB}
\]
Design example 4: a CCM forward converter

- Build an average model to extract \( H(s) \)

Parameters

- \( V_{in}=5 \)
- \( L_{in}=5u \)
- \( L_{mag}=10mH \)
- \( F_s=200k \)
- \( N=0.5 \)
- \( R_{sense}=70\Omega \)
- \( R_{load}=250\Omega \)
- \( V_{in}=36 \)

\[ \text{Vinmin}=36 \]
\[ D=0.31 \]
\[ \text{Smag}=(\text{Vinmin}\cdot\text{Vin})/\text{Rsense} \]
\[ \text{Sn}=(N\cdot\text{Vinmin}\cdot\text{Vin})/N/\text{Rsense} \]
\[ Q=1/(\pi\cdot(1-D-0.5)) \]
\[ mc=((1/\pi)+0.5)/(1-D) \]
\[ \text{Se}=(mc-1)\cdot\text{Sn}-\text{Smag} \]

Ramp compensation calculations

- \( V_{in}=36 \)
- \( D=0.31 \)
- \( \text{Samp}=5\cdot\text{Vin}\cdot\text{Vin}/\text{Rsense} \)
- \( \text{Sn}=(N\cdot\text{Vinmin}\cdot\text{Vin})/N/\text{Rsense} \)
- \( \text{Q}=1/(\pi\cdot(1-D-0.5)) \)
- \( \text{mc}=(1/\pi)+0.5 \)
- \( \text{Se}=(mc-1)\cdot\text{Sn}-\text{Samp} \)

Design example 4: a CCM forward converter

- From the power stage Bode plot, extract the data at \( f_c \)

Place:

- \( f_c = 6.8\ kHz \)
- \( f_r = 14.5\ kHz \)

\[ |H(z)| \]

\[ \angle H(z) \]

\[ \angle H(10k) = -51^\circ \]

\[ H(10k) = -17.2\ dB \]

\( V_{in}=36\ V \)

\( 10 \quad 100 \quad 1k \quad 10k \quad 100k \)
Design example 4: a CCM forward converter

- Check the impact on parameters such as CTR, ESR etc.

- The CTR variation induces an upper crossover of 23 kHz
- This is an aggressive target, prone to collecting noise
- Better reduce the initial crossover to limit $f_c$ to $\approx 15$ kHz
Design example 4: a CCM forward converter

- The opto wired to the ground, the fast lane goes away

- The control phase is reversed, watch for the right polarity!

Agenda

- Feedback generalities
- The divider and the virtual ground
- Phase margin and crossover
- Poles and zeros
- Boosting the phase at crossover
- Various compensator types
- Practical implementations: the op amp
- Practical implementations: the OTA
- Practical implementations: the TL431
- Design examples
- A real case study
- Conclusion
A real-case example with a UC384X

- A 19-V/3-A converter is built around an UC3843

- The converter operates in CCM at full load low line

A real-case example with a UC384X

- Use an auto-toggling current-mode average model

1 V maximum voltage and divider by 3 CCM operation Low line voltage Dc + ac modulation
A real-case example with a UC384X

- $H(s)$ alone can be measured without loop opening

Watch out for capacitor connection (short-circuit to GND when discharged)

A real-case example with a UC384X

- For closed-loop measurements, a transformer is the solution

- Make sure $Z_{\text{out}} \ll Z_{\text{in}}$ to avoid gain errors
A real-case example with a UC384X

CCM operation, $R_{\text{load}} = 6.3 \, \Omega$

A real-case example with a UC384X

DCM operation, $R_{\text{load}} = 20 \, \Omega$
A real-case example with a UC384X

- Select the crossover point on the open-loop Bode plot

- The TL431 is tailored to pass a 1-kHz bandwidth
  
  - Calculate mid-band gain: +18 dB
  
  \[ R_{\text{pulldown}} = \frac{R_{\text{pulldown}} \cdot \text{CTR}}{10^{31}} = \frac{4.7 \times 0.45}{7.94} = 266 \Omega \]

  - We place a zero at 300 Hz:
    \[ C_{\text{zero}} = \frac{1}{2 \pi f_{\text{zero}} R_{\text{pulldown}}} = \frac{1}{6.28 \times 300 \times 66} = 8 \text{nF} \]

  - We place a pole at 3.3 kHz:
    \[ C_{\text{pole}} = \frac{1}{2 \pi f_{\text{pole}} R_{\text{pulldown}}} = \frac{1}{6.28 \times 3.3 \times 4.7} = 10 \text{nF} \]

*“Switch-Mode Power Supplies: SPICE Simulations and Practical Designs”, McGraw-Hill*
A real-case example with a UC384X

- Sweep extreme voltages and loads as well!

Simulated CCM operation, $R_{\text{load}} = 6.3 \, \Omega$, $V_{\text{in}} = 150 \, \text{Vdc}$

Simulated CCM operation, $R_{\text{load}} = 6.3 \, \Omega$, $V_{\text{in}} = 330 \, \text{Vdc}$
A real-case example with a UC384X

DCM operation, \( R_{\text{load}} = 20 \, \Omega \), \( V_{\text{in}} = 330 \, \text{Vdc} \)

Good agreement between curves!

\( V_{\text{in}} = 150 \, \text{V} \)
CCM
2 to 3 A
1 A/µs
A real-case example with a UC384X

- DCM operation at high line is also stable

Conclusion

- We have seen how to apply loop theory to a switching converter
- Classical type 1, 2 and 3 compensators have been covered
- Their implementation with op amps, OTAs and TL431 studied
- Op amps are the most flexible, OTAs and TL431 have limits
- In isolated supplies, the optocoupler affects the transmission chain
- Design examples showed the power of averaged models
- Use them to extensively test the loop stability (sweep ESRs etc.)
- Applying these recipes is key to design success!