Small-Signal Modeling at Work with Power Converters

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Course Agenda

- Introducing the PWM Switch Model
- CCM, DCM and BCM in Voltage Mode
- Pulse Width Modulator Gain
- The PWM Switch Model in Current Mode
- PWM Switch at Work in a Buck Converter
- A Simplified Approach to Modeling a DCM Boost
- Transfer Function of a BCM Boost in Current Mode
- Small-Signal Model of The Active Clamp Forward
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Manipulating Linear Networks

- A switching converter is made of linear elements!

- The non-linearity or discontinuity is coming from transitions

\[ V_{in} \quad r_{DS(on)} \quad L \quad r_L \quad V_{out} \quad C \quad R_{load} \]

\[ V_{in} \quad r_d \quad L \quad r_L \quad V_{out} \quad C \quad R_{load} \]

Cannot differentiate

Singularity

Linear

Linear

Linear

\[ v_{DRV}(t) \]

\[ DT_{sw} \]

\[ (1-D)T_{sw} \]
State Space Averaging (SSA)

- Despite linear networks, equation is discontinuous in time
- Introduced in 76, SSA weights on and off expressions

\[
\dot{x} = \underbrace{\left[ A_1 D + A_2 (1-D) \right]}_{\text{weighted during } (1-D)T_{sw}} x(t) + \underbrace{\left[ B_1 D + B_2 (1-D) \right]}_{\text{weighted during } DT_{sw}} u(t)
\]

- This equation is now continuous in time: singularity is gone
- However, it became a non-linear equation
  - You need to linearize it by perturbation (or differentiation)
  - If you add a new element, you have to restart from scratch!

The PWM Switch Model in Voltage Mode

- We know that non-linearity is brought by the switching cell.

![Switching cell diagram](image)

- Why don't we linearize the cell alone?

![Small-signal model diagram](image)

The Bipolar Small-Signal Model

- A bipolar transistor is a highly non-linear system
- Replace it by its small-signal model to get the response

Ebers-Moll model
Replace the Switches by the Model

- Like in the bipolar circuit, replace the switching cell…

- ...and solve a set of linear equations!
An Invariant Model

- The switching cell made of two switches is everywhere!

![Diagrams showing buck, buck-boost, boost, and Ćuk circuits]
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CCM, DCM and BCM Operations

Three types of conduction modes exist:

1. Continuous Conduction Mode
   \[ \langle i_L(t) \rangle_{T_{sw}} > \frac{I_{peak}}{2} \]

2. Discontinuous Conduction Mode
   \[ \langle i_L(t) \rangle_{T_{sw}} < \frac{I_{peak}}{2} \]

3. Boundary Conduction Mode
   \[ \langle i_L(t) \rangle_{T_{sw}} = \frac{I_{peak}}{2} \]

Each mode has its own small-signal characteristics.

A model is needed for these three modes!
CCM Common Passive Configuration

- The PWM switch is a single-pole double-throw model

![Diagram of PWM switch with labels](attachment: PWM_switch_diagram.png)

- Install it in a buck and draw its terminals waveforms

![Diagram of buck converter with labels and waveforms](attachment: buck_converter_diagram.png)

CCM VM
The Common Passive Configuration

- Average the current waveforms across the PWM switch

\[ i_c(t) \]

\[ i_a(t) \]

\[ \langle i_c(t) \rangle_{T_{sw}} \]

\[ \langle i_a(t) \rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}} \int_0^{D T_{sw}} i_a(t) \, dt = D \langle i_c(t) \rangle_{T_{sw}} = DI_c \]
The Common Passive Configuration

- Average the voltage waveforms across the PWM switch

\[ v_{ap}(t) \]
\[ v_{cp}(t) \]

\[ \langle v_{cp}(t) \rangle_{T_{sw}} = V_{cp} = \frac{1}{T_{sw}} \int_{0}^{D_{T_{sw}}} v_{cp}(t) \, dt = D \langle v_{ap}(t) \rangle_{T_{sw}} = DV_{ap} \]

Averaged variables

\[ V_{cp} = DV_{ap} \]
A Two-Port Representation

- We have a link between input and output variables

\[
\begin{align*}
V_{ap} & \quad \quad DI_c & \quad \quad V_{ap} \\
I_c & \quad \quad DV_{ap} & \quad \quad I_c \\
\end{align*}
\]

- It can further be illustrated with current and voltage sources

\[
\begin{align*}
V_{ap} & \quad \quad I_a & \quad \quad DI_c & \quad \quad DV_{ap} & \quad \quad I_c & \quad \quad V_{cp} \\
\end{align*}
\]
A Transformer Representation

- The PWM switch large-signal model is a dc "transformer"!

\[
\begin{align*}
I_a &= DI_c \\
I_c &= \frac{I_a}{D}
\end{align*}
\]

dc equations!

- It can be plugged into any 2-switch CCM converter

Dc bias point

Ac response

CCM VM
The Discontinuous Case

In DCM, a third timing event exists when $i_L(t) = 0$

$$i_L(t)$$

$V_{in}$

$SW$ $L$ $C$ $R$

During $D_1 T_{sw}$

During $D_2 T_{sw}$

During $D_3 T_{sw}$

$DCM VM$
The Same Configuration as in CCM

- Draw the waveforms in the "common passive" configuration

\[ i_a(t) \]
\[ v_{ap}(t) \]
\[ i_c(t) \]
\[ v_{cp}(t) \]

\[ I_{peak} \]
\[ V_{in} \]
\[ V_{out} \]

Average the waveforms:

\[ I_a = \frac{I_{peak}}{2} D_1 \]

\[ I_c = \frac{I_{peak}}{2} D_1 + \frac{I_{peak}}{2} D_2 = \frac{I_{peak}}{2} (D_1 + D_2) \]

\[ I_c = \frac{2I_a}{D_1} \frac{D_1 + D_2}{2} = I_a \frac{D_1 + D_2}{D_1} \]

DCM VM
Derive $V_{cp}$ to Unveil the New Model

- The addition of the third event complicates the equations

$$V_{cp} = V_{ap} D_1 + V_{cp} D_3$$

$$D_1 + D_2 + D_3 = 1$$

$$V_{cp} = V_{ap} D_1 + V_{cp} (1 - D_1 - D_2)$$

$$V_{cp} = V_{ap} \frac{D_1}{D_1 + D_2}$$

$$I_a = N_{dc} I_c$$

$$I_c = \frac{I_a}{N_{dc}}$$

$$N_{dc} = \frac{D_1}{D_1 + D_2}$$

$$f(D_1)$$

$$V_{ap} = \frac{V_{cp}}{N_{dc}}$$

$$V_{cp} = N_{dc} V_{ap}$$

DCM VM
Finally, Get the $D_2$ Value

- In DCM the inductor average voltage per cycle is always 0

$$V_{cp} = V_{out}$$

- What is the averaged inductor peak current?

$$I_{peak} = \frac{\langle v_L(t) \rangle_{D_1T_{sw}}}{L} D_1T_{sw}$$

$$\langle v_L(t) \rangle_{D_1T_{sw}} = V_{ac}$$

$$V_{ac} = L \frac{I_{peak}}{D_1T_{sw}}$$

- The peak current uses a previous expression

$$I_c = \frac{I_{peak}}{2}(D_1 + D_2)$$

$$I_{peak} = \frac{2I_c}{D_1 + D_2}$$

$$D_2 = \frac{2LF_{sw}}{D_1} \frac{I_c}{V_{ac}} - D_1$$
Variable Frequency Operation

- The power supply operates in a self-oscillating mode

**Diagram:**
- Power supply operates in Critical Conduction Mode (CRM)
- Boundary Conduction Mode (BCM)
- Demag detector
- Demag detector
- CTR
- FB
- $V_{dd}$
- $R_{pullup}$
- $G_{FB}$
- $R_{sense}$
- $C_{out}$
- $R_{load}$
- $V_{bulk}$
- $V_{out}$
- $N_p:N_s$
- $L_p$
- 65 mV

**Equations:**
- $V_{dd}$
- $R_{pullup}$
- $G_{FB}$
- $R_{sense}$
- $C_{out}$
- $R_{load}$
- $V_{bulk}$
- $V_{out}$
- $N_p:N_s$
- $L_p$
- 65 mV
Benefits of Quasi-Resonance

- Wait for core demagnetization and valley voltage
Idealized Waveforms

- Draw the PWM switch waveforms in a buck configuration

\[ i_a(t) \]

\[ i_c(t) \]

\[ v_{cp}(t) \]

\[ I_{peak} \]

\[ \left\langle i_c(t) \right\rangle_{T_{sw}} \]

\[ V_{ap} \]

BCM VM
Derive Founding Equations

- Average current in terminal C is straightforward

\[ \langle i_c(t) \rangle_{T_{sw}} = \frac{I_{peak}}{2} \]

- The off time depends on the voltage across the inductor

\[ t_{off} = \frac{L}{V_{cp}} I_{peak} \quad I_{peak} = 2I_c \quad \rightarrow \quad t_{off} = \frac{2LI_c}{V_{cp}} \]

- Period and duty ratio come easily as \( t_{on} \) is imposed

\[ t_{on} + t_{off} = T_{sw} \quad D = \frac{t_{on}}{T_{sw}} \]

Need to transform the error voltage into time
Generating the On Time

- The on time modulator works as a PWM block

\[ t_{on}(V_{err}) = \frac{V_{err}C}{I} \]

- The modulator small-signal gain is a simple coefficient

\[ \frac{d}{dV_{err}} t_{on}(V_{err}) = \frac{C}{I} \]

\[ C = 100 \text{pF} \quad I = 20 \mu\text{A} \quad \frac{C}{I} = 5u \]

Assuming 1 V = 1 \mu s

\[ V_{err} \rightarrow 5 \rightarrow t_{on} \]

BCM VM
Final PWM Switch Model in BCM

- Just add the on-time modulator to the VM PWM switch

\[ V_{err} \rightarrow k = \frac{C}{I} \rightarrow t_{on} \]

\[ V_{ac} \rightarrow I_{\text{peak}} = \frac{V_{ac} t_{on}}{L} \rightarrow I_{\text{peak}} \]

\[ V_{cp} \rightarrow t_{off} = \frac{L}{V_{cp}} I_{\text{peak}} \rightarrow t_{off} \]

\[ D = \frac{t_{on}}{t_{on} + t_{off}} \]

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What About the PWM Block?

- In voltage mode, the duty ratio depends on $V_{err}$

\[
\begin{align*}
V_{err}(t) &= V_p \frac{t_{on}(t)}{T_{sw}} \\
D(t) &= \frac{t_{on}(t)}{T_{sw}} \\
\frac{D(t)}{V_{err}(t)} &= \frac{1}{V_p} \\
\frac{d}{dV_{err}} D(V_{err}) &= G_{PWM} = \frac{1}{V_p}
\end{align*}
\]
Including the PWM Contribution

- In a simulation fixture, insert a gain block after $V_{err}$

$$G_{PWM} = \frac{1}{V_p}$$

Modulator gain

PWM switch model in VM
Considering Feedforward

- A perturbation disturbs operations and must be fought
- In a power supply, the input voltage is a perturbation

- The perturbation must affect the output to trigger action
  - Why not reacting before perturbation reaches the output?
  - This is the principle of feedforward
Input Contribution in a Buck Converter

- The transfer function of a CCM buck converter includes $V_{in}$:

$$H(s) = \frac{V_{in}}{V_p} \cdot \frac{1 + \frac{s}{s_{z1}}}{1 + \frac{s}{\omega_0Q} + \left(\frac{s}{\omega_o}\right)^2}$$

- Let's make $V_p$ a function of $V_{in}$: $V_p(V_{in}) = k_{FF}V_{in}$

$$H(s) = \frac{V_{in}}{k_{FF}V_{in}} \cdot \frac{1 + \frac{s}{s_{z1}}}{1 + \frac{s}{\omega_0Q} + \left(\frac{s}{\omega_o}\right)^2} = \frac{1}{k_{FF}} \cdot \frac{1 + \frac{s}{s_{z1}}}{1 + \frac{s}{\omega_0Q} + \left(\frac{s}{\omega_o}\right)^2}$$

- The transfer function no longer depends on $V_{in}$.
How to Make $V_p$ a Function of $V_{in}$?

- Make the sawtooth capacitor current depend on $V_{in}$

![Diagram]

$V_{err}$

$R$

$I_C$

$C$

$V_{in}$

PWM

$v_c(t)$

$V_{err}$

$t$

HL = hi line

LL = lo line
Ramp Amplitude and Input Voltage

- We can neglect the sawtooth ramp amplitude
  \[ I_c \approx \frac{V_{in}}{R} \]

- The peak value, \( V_p \), is linked to the time constant \( \tau \)
  \[ V_p = \frac{I_c}{C_{ramp} T_{sw}} = \frac{V_{in}}{C_{ramp} R_{ramp} T_{sw}} = \frac{V_{in}}{\tau F_{sw}} \]

- In the time domain, the peak value will change
  \[ v_{ramp}(t) = V_p \frac{t}{T_{sw}} = \frac{V_{in}}{\tau F_{sw}} \frac{t}{T_{sw}} \]

- At \( t = t_{on} \), the error voltage equals \( V_{ramp} \)
  \[ V_{err} = \frac{V_{in}}{\tau F_{sw}} \frac{t_{on}}{T_{sw}} = \frac{V_{in}}{\tau F_{sw}} D \quad \rightarrow \quad D(V_{err}) = V_{err} \frac{\tau F_{sw}}{V_{in}} \]
An In-Line Equation to Include Feedforward

- Differentiate the expression to get the small-signal gain

\[ G_{PWM} = \frac{\partial D(V_{err})}{\partial V_{err}} = \frac{\tau F_{sw}}{V_{in}} = \frac{1}{k_{FF} V_{in}} \]

\[ k_{FF} = \frac{1}{F_{sw} \tau} \]

- The feedforward block requires an ABM source

\[ R = 82 \text{ k} \Omega \]
\[ C = 470 \text{ pF} \]
\[ F_{sw} = 500 \text{ kHz} \]

\[ k_{FF} = \frac{1}{500k \times 470p \times 82k} = 52 \text{m} \]

ABM: Analog Behavioral Model
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Peak Current Mode Control

- In voltage-mode, the loop controls the duty ratio
- In current-mode, the inductor peak current is controlled

An artificial ramp is added for stabilization purposes
We Want the Average Current Definition

- The value $I_c$ is the inductor current at half the ripple

\[ I_c(t) \]

\[ I_{\text{peak}} \]

\[ \langle I_c \rangle_{T_{sw}} = \frac{V_c}{R_i} - \frac{S_a}{R_i} DT_{sw} - \frac{S_2 D'T_{sw}}{2} \]

- Current at point b is that of a minus half the inductor ripple

CCM CM
Define the Converter off-Slope

- Use a buck configuration to see voltages at play

The downslope depends on the output voltage $V_{out}$:

$$S_2 = -\frac{V_{out}}{L}$$

- The inductor average voltage is 0 at steady-state

$$V_{cp} = V_{out} \quad \rightarrow \quad S_2 = -\frac{V_{cp}}{L}$$
A Current Mode Generator

- Update the previous equation to obtain final definition

$$I_c = \frac{V_c}{R_i} - \frac{V_{cp} (1 - D) T_{sw}}{2L} - \frac{S_a D T_{sw}}{R_i}$$

Peak current setpoint  Half inductor ripple  Compensation ramp

- Inductor ripple and compensation ramp alter peak value

$$I_\mu = V_{cp} (1 - D) \frac{T_{sw}}{2L} + \frac{S_a D T_{sw}}{R_i}$$

Group 2\textsuperscript{nd} and 3\textsuperscript{rd} terms
CM or VM Lead to Similar Input Currents

- Average the current waveforms across the PWM switch

\[ \langle i_c(t) \rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}} \int_0^{dT_{sw}} i_a(t) \, dt = D \langle i_c(t) \rangle_{T_{sw}} = DI_c \]

\[ I_a = DI_c \]

\[ I_a = \frac{V_{cp}}{V_{ac}} I_c \]
The PWM Switch Model in Current Mode

- The final model associates three current sources

This is the large-signal current-mode PWM switch model

V. Vorpérian, "Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch", PCIM Conference, 1990
Final Model Includes Subharmonic Effects

- A simple capacitor is enough to mimic instability

![Circuit Diagram]

- As the instability is placed at half the switching frequency:

\[
\frac{F_{sw}}{2} = \frac{1}{2\pi\sqrt{LC_s}} \quad \Rightarrow \quad C_s = \frac{1}{L\left(F_{sw}\pi\right)^2}
\]
The PWM CM can work in discontinuous mode

The average current $I_c$ is somewhere in the downslope $S_2$

$$I_{\text{peak}} = \frac{V_c - D_1 T_{sw} S_a}{R_i}$$

$$I_c = \frac{V_c - D_1 T_{sw} S_a}{R_i} - \alpha D_2 T_{sw} S_2$$
Derive the Inductor Average Current

- We must now obtain the value of $\alpha$ to get $I_c$

$$I_{\text{peak}} - I_c \left\{ \begin{array}{c}
S_2 \end{array} \right\} \alpha I_{\text{peak}}$$

$I_c$ is the area under the triangle divided by the switching period

$$I_c = \frac{I_{\text{peak}} D_1}{2} + \frac{I_{\text{peak}} D_2}{2} = I_{\text{peak}} \frac{D_1 + D_2}{2}$$

$$\alpha I_{\text{peak}} = I_{\text{peak}} - I_{\text{peak}} \frac{D_1 + D_2}{2}$$

$$\alpha = 1 - \frac{D_1 + D_2}{2}$$
Adopt the CCM Structure for DCM

- Substitute and rearrange to get the inductor current

\[
I_c = \frac{V_c}{R_i} - \frac{D_1 T_{sw} S_a}{R_i} - D_2 T_{sw} \frac{V_{cp}}{L} \left(1 - \frac{D_1 + D_2}{2}\right)
\]

- If we stick to the original CCM architecture

\[
I_c = \frac{V_c}{R_i} - I_\mu \quad \text{with} \quad I_\mu = \frac{D_1 T_{sw} S_a}{R_i} + D_2 T_{sw} \frac{V_{cp}}{L} \left(1 - \frac{D_1 + D_2}{2}\right)
\]
Discontinuous Waveforms

Let's have a look at the PWM switch voltages in DCM

\[ i_a(t) \]

\[ v_{ap}(t) \]

\[ v_{cp}(t) \]

\[ i_c(t) \]

\[ V_{in} \]

\[ V_{out} \]

\[ V_{ap} \]

\[ I_{peak} \]

\[ \langle i_a(t) \rangle_{T_{sw}} \]

\[ D_1 T_{sw} \]

\[ D_2 T_{sw} \]

\[ D_3 T_{sw} \]

DCM
Derive the Duty Ratios

- From the DCM voltage-mode PWM switch we have:

\[ V_{cp} = V_{ap} \frac{D_1}{D_1 + D_2} \quad \Rightarrow \quad D_1 = \frac{D_2 V_{cp}}{V_{ap} - V_{cp}} \]

- From the operating DCM waveforms:

\[ I_a = \frac{I_{peak} D_1}{2} \quad \Rightarrow \quad I_{peak} = \frac{2I_a}{D_1} \quad \Rightarrow \quad I_c = \frac{I_{peak}}{2} \quad \Rightarrow \quad I_a = I_c \frac{D_1}{D_1 + D_2} \]

- Almost there, just need to express \( D_2 \):

\[ I_{peak} = \frac{V_{ac}}{L} D_1 T_{sw} \quad \text{and} \quad I_{peak} = \frac{2I_c}{D_1 + D_2} \]

\[ \frac{V_{ac} D_1 T_{sw}}{L} = \frac{2I_c}{D_1 + D_2} \quad \Rightarrow \quad D_2 = \frac{2LF_{sw}}{D_1} \frac{I_c}{V_{ac}} - D_1 \]

DCM CM
The DCM Model is Complete!

- We can use this model for DCM simulations

\[
I_a = I_c \frac{D_1}{D_1 + D_2} \quad \quad I_\mu = \frac{D_1 T_{sw} S_a}{R_i} + D_2 T_{sw} \frac{V_{cp}}{L} \left( 1 - \frac{D_1 + D_2}{2} \right)
\]
Current Mode Borderline

- In CM, the error voltage sets the peak current

\[ I_{\text{peak}} = \frac{V_c}{R_i} \]

- The peak current also depends on duty ratio \( D \)

\[ I_{\text{peak}} = \frac{V_{ac}}{L} DT_{\text{sw}} \]

\[ D = \frac{V_c}{R_i} \frac{L}{V_{ac} T_{\text{sw}}} \]

\[ t_{\text{on}} = \frac{V_c}{R_i} \frac{L}{V_{ac}} \]
Operating Points of BCM Current Mode

- The off-time duration depends on $I_{peak}$ too

$$I_{peak} = \frac{V_{cp}}{L}(1 - D)T_{sw} \quad t_{off} = \frac{V_{err} L}{R_i V_{cp}}$$

- Switching frequency comes easily

$$T_{sw} = t_{on} + t_{off} = \frac{V_c L}{R_i} \left( \frac{1}{V_{ap}} + \frac{1}{V_{cp}} \right)$$

- The average inductor current $I_c$ is straightforward

$$I_c = \frac{I_{peak}}{2} = \frac{V_c}{2R_i}$$

- The relationship between $I_a$ and $I_c$ is always the same

$$I_a = DI_c$$
This Completes the BCM CM Model

The model is really simple, two current sources

Other ABM sources will compute $D$ and $T_{sw}$
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A Buck Converter in Current Mode

- Identify the diode and switch position in a buck CM

- Replace switches by the small-signal PWM switch model
A Small Signal Model

- The model includes current sources and conductances

\[
\begin{align*}
  k_o &= \frac{1}{R_i} \\
  g_f &= Dg_o - \frac{DD'T_{sw}}{2L} \\
  g_o &= \frac{T_{sw}}{L} \left( D' \frac{S_a}{S_n} + \frac{1}{2} - D \right) \\
  k_i &= \frac{D}{R_i} \\
  g_i &= D \left( g_f - \frac{I_c}{V_{ap}} \right) \\
  g_r &= \frac{I_c}{V_{ap}} - g_o D
\end{align*}
\]

V. Vorpérian, "Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch", PCIM Conference, 1990
Plug the Model and Simplify

Plug, simplify and rearrange: test in between!

We want the control-to-output function, remove input stimulus

\[ \hat{v}_{in} = 0 \quad \rightarrow \quad \hat{v}_{ap} g_f = 0 \]

The input contribution can also disappear, no interest in \( Z_{in} \)
Time to Call FACTS!

- End-up with a simpler and less ugly sketch

- There are 3 storage elements: 3rd-order system

FACTS: Fast Analytical Circuit TechniqueS
Don't Use Brute-Force Algebra!

- You can use brute-force analysis...

\[
V_{th} = V_c(s)k_0 \left( \frac{1}{g_o} \parallel \frac{1}{sC_s} \right)
\]

\[
Z_{th} = \left( \frac{1}{g_o} \parallel \frac{1}{sC_s} \right)
\]

- …or consider FACTS to write a 3rd-order system TF

\[
H(s) = H_0 \frac{N(s)}{D(s)} = H_0 \frac{N(s)}{1 + a_1s + a_2s^2 + a_3s^3}
\]
Start with the Dc Gain

- Consider dc and high-frequency states for $L$ and $C$

  - DC state: $Z_C = \infty$ (Cap. is an open circuit)
  - HF state: $Z_C = 0$ (Cap. is a short circuit)

  - DC state: $Z_L = 0$ (Inductor is a short circuit)
  - HF state: $Z_L = \infty$ (Inductor is an open circuit)

- In the circuit, open capacitors and short inductors

\[ V_c(s)k_o \quad 1 \quad R \quad \frac{1}{g_o} \quad \rightarrow \quad H_0 = k_o \left( R \parallel \frac{1}{g_o} \right) \]

10 s!
Identify The Zeros

- Zeros prevent the excitation from reaching the output

\[ V_{in}(s) \quad Z(s) = \infty \quad V_{out}(s) \]

excitation

- We have the zero expression, half of the work is done

\[ H(s) = H_0 \frac{s_{z_1}}{1 + a_1s + a_2s^2 + a_3s^3} \]
Finding the Poles

- The poles are linked to the time constants of the system
- These time constants solely depend on the structure

→ Remove the excitation signal to isolate the structure

- The denominator order depends on the storage elements

1 storage element
1\textsuperscript{st}-order

2 storage elements
2\textsuperscript{nd}-order

Not always! Consider individual state variables.
Start by Identifying the Time Constants

- The excitation is zero, elements are in their dc states
- 3 storage elements, 3 time constants, 3 drawings

1. \( \tau_1 = f(C_s) \Rightarrow \tau_1 = C_s \left( \frac{1}{g_0} + R \right) \)

2. \( \tau_2 = f(L) \Rightarrow \tau_2 = \frac{L}{1/g_0 + R} \)

3. \( \tau_3 = f(C) \Rightarrow \tau_3 = C \left( r_C + \frac{1}{g_0} + R \right) \)
First Coefficients $a_1$ and $a_2$

- FACTs tell us that $a_1$ sums up all time constants
  \[ a_1 = \tau_1 + \tau_2 + \tau_3 \quad \rightarrow \quad \text{Dimension is time} \]

- For $a_2$, we multiply combined-time constants
  \[ a_2 = \tau_1 \tau_2^{1} + \tau_1 \tau_3^{1} + \tau_2 \tau_3^{2} \quad \rightarrow \quad \text{Dimension is time}^2 \]

- What is this new time constants definition, $\tau_2^{1}$?

\[
\begin{array}{c|c|c|c}
\tau_2^{1} & C_s \text{ (HF)} & \tau_3^{1} & C_s \text{ (HF)} & \tau_3^{2} & L \text{ (HF)} \\
\hline
L & C \text{ (dc)} & L \text{ (dc)} & C & C & C_s \text{ (dc)} \\
\uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
\end{array}
\]

Coefficient $a_2$ Mixes Time Constants

- Drawings are key to avoid mistakes

\[
\begin{align*}
\tau_2^1 &= \frac{L}{R} \\
\tau_3^1 &= r_C C
\end{align*}
\]
(Carefully) Mixing Time Constants

- The last drawing completes the $a_2$ expression

$$
\tau_3^2 = \frac{1}{g_0} \left( r_c + R \right) C
$$

- $a_2$ coefficient is there!

$$
a_2 = C_s \left( \frac{1}{g_o} \parallel R \right) \frac{L}{R} + C_s \left( \frac{1}{g_o} \parallel R \right) r_c C + \frac{L}{\frac{1}{g_0} + R} \left( r_c + R \right) C
$$
...And $a_3$ is?

- For $a_3$, we multiply by a third time-constant
  \[ a_3 = \tau_1 \tau_2 \tau_3 \]  
  Dimension is time$^3$

- What is this new time constant definition?

- The final coefficient has been identified
  \[ a_3 = C_s \left( \frac{1}{g_o} \| R \right) \frac{L}{R} (r_C + R) C \]
Time to Check Results

A Mathcad® sheet can be built to verify these calculations

\[ H(s) = G_0 \frac{1 + \frac{s}{S_{z_1}}}{1 + a_1 s + a_2 s^2 + a_3 s^3} \]

\[ G_0 = k_0 \left( R \left| \frac{1}{g_0} \right| \right) \]

\[ \omega_{z_1} = \frac{1}{r_C C} \]

\[ a_1 = C_s \left( \frac{1}{g_o} \right) + \frac{L}{1 + R} + C \left( r_C + \left( \frac{1}{g_o} \right) \right) \]

\[ a_2 = C_s \left( \frac{1}{g_o} \right) \frac{L}{R} + C_s \left( \frac{1}{g_o} \right) r_C C + \frac{L}{1 + R} (r_C + R) C \]

\[ a_3 = C_s \left( \frac{1}{g_o} \right) \frac{L}{R} (r_C + R) C \]

5 V/1 A buck

\[ V_{in} = 10 \text{ V}, F_{sw} = 100 \text{ kHz}, R_i = 0.25 \Omega, S_e = 2.5 \text{ kV/s} \]

\[ C = 100 \mu \text{F}, r_C = 0.1 \Omega, L = 100 \mu \text{H}, C_s = 101 \text{nF}, V_c = 1.28 \text{ V} \]

\[ I_c = 4.94 \text{ A} \]

\[ k_i = 2 \Omega^{-1} \quad k_0 = 4 \Omega^{-1} \]

\[ g_0 = 0.01 \Omega^{-1} \]

\[ g_f = -7.5 \text{ m} \Omega^{-1} \quad g_r = 0.49 \Omega^{-1} \]

\[ g_i = -250 \text{ m} \Omega^{-1} \]

\[ G_0 = 12 \text{ dB} \quad f_{z_1} = 15.9 \text{ kHz} \]
See What SPICE is Saying

- Use the large-signal model, SPICE linearizes it for you

Then compare results with those of Mathcad®
Excellent Agreement!

- Superimposed curves mean transfer functions are identical
Rearranging Expressions

- The denominator is not really in a low-entropy form

\[ D(s) = 1 + a_1s + a_2s^2 + a_3s^3 \]

- This is a third-order polynomial form that can be factored

\[
D(s) \approx 1 + \frac{a_2}{a_1} s + \frac{a_3}{a_1} s^2
\]
Final Lap!

- The transfer function can now unveil peaking and damping

\[
H(s) \approx H_0 \frac{1 + \frac{s}{\omega_{z_1}}}{1 + \frac{s}{\omega_{p_1}} + \frac{s}{\omega_n Q} + \left(\frac{s}{\omega_n}\right)^2}
\]

\[
H_0 = \frac{R}{R_i} \frac{1}{1 + \frac{RT_{sw}}{L} \left[ m_c (1 - D) - 0.5 \right]}
\]

\[
\omega_{p_1} = \frac{1}{RC} + \frac{T_{sw}}{LC} \left[ m_c (1 - D) - 0.5 \right]
\]

\[
\omega_n = \frac{\pi}{T_{sw}} \quad Q = \frac{1}{\pi \left[ m_c (1 - D) - 0.5 \right]}
\]

\[
m_c = 1 + \frac{S_e}{S_n}
\]

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A DCM Boost Converter

- The goal is to regulate the LED string current

- The LED current is sensed via a shunt element
Characterize the LED String

- Evaluate forward drop and dynamic resistance

\[ v_f = I_F r_d + V_{T_0} \]

- Lab. measurements require simple V and A-meters

\[ r_{LEDs} = \frac{V_{f_1} - V_{f_2}}{I_{F_1} - I_{F_2}} = \frac{27.5 - 26.4}{0.1 - 0.08} = 55 \Omega \]

\[ V_Z \approx V_{f_1} - R_{LEDs} I_{F_1} = 27.5 - 0.1 \times 55 = 22 \text{ V} \]
A Simplified Approach

- The LED string is driven by a current source

\[ R_{ac} = r_{LEDs} + R_{sense} \]
\[ V_{out} = R_{ac}I_{out} + V_z \]
\[ V_{out}(s) = R_{ac}I_{out}(s) \]

- \( V_z \) sets the operating point, \( R_{ac} \) sets the ac response
Current Source Model

- It is convenient and fast to consider 1\textsuperscript{st}-order models
- Use the converter transfer function in DCM

\[
\frac{V_{out}}{V_{in}} = 1 + \sqrt{1 + \frac{2T_{sw}D^2R_{dc}}{L}}
\]

- We purposely consider an instantaneous power response
  - if \( D \) changes, \( P_{out} \) immediately translates
  - this is not true for boost or buck-boost converters: RHPZ
  - high-frequency phenomena are also lost
Define the Duty Ratio

- In the previous expression, $D$ is unknown

\[
\frac{v_c(t)}{R_i} = \frac{S_e}{R_i} I_{peak} \quad \frac{S_n}{L} = \frac{V_{in}}{L} \quad i_D(t)
\]

\[
I_{peak} = \frac{V_c}{R_i} - \frac{S_e}{R_i} DT_{sw}
\]

\[
I_{peak} = \frac{DT_{sw} V_{in}}{L}
\]

\[
D = \frac{V_c L}{S_e T_{sw} L + R_i T_{sw} V_{in}}
\]
Update the Output Current Equation

- Massage the dc transfer equation to unveil $I_{out}$

$$\frac{V_{out}}{V_{in}} = 1 + \frac{1}{1 + \frac{L}{2}} \sqrt{\frac{T_{sw} D^2 V_{out}}{I_{out}}}$$

- Inject the duty ratio definition, solve for $I_{out}$

$$I_{out} = \frac{2V_{out}L V_c^2}{T_{sw}} \frac{1}{\left(\frac{2V_{out}}{V_{in}} - 1\right)^2 - 1} \left(S_e L + R_i V_{in}\right)^2$$

- There are two modulated variables, $V_c$ and $V_{out}$

$$\hat{I}_{in} = 0$$

$$\begin{align*}
\frac{\partial I_{out}}{\partial V_c} &\bigg|_{\hat{V}_c} = \hat{V}_c \\
\frac{\partial I_{out}}{\partial V_{out}} &\bigg|_{\hat{V}_c} = \hat{V}_{out}
\end{align*}$$
A Simple Model

- You obtain two small-signal sources: \( \hat{i}_{out} = g_1 \hat{v}_c + g_2 \hat{v}_{out} \)

\[
g_1 = \frac{\partial I_{out}(V_c, V_{out})}{\partial V_c} = \frac{V_{in}^2 V_c L}{T_{sw}(V_{out} - V_{in})(S_e L + R_i V_{in})^2}
\]

\[
g_2 = \frac{\partial I_{out}(V_c, V_{out})}{\partial V_{out}} = -\frac{V_{in}^2 V_c^2 L}{2T_{sw}(V_{in} - V_{out})^2 (S_e L + R_i V_{in})^2}
\]

- The circuit is quite simple.
A Current Driving an Impedance

- The second term is a simple resistance

\[ I = \frac{V}{R} \]

\[ R_1 = \frac{2T_{sw} (V_{in} - V_{out})^2 (S_L + R_i V_{in})^2}{V_c^2 V_{in}^2 L} \]

- Update the final model

\[ V_{out}(s) = V_c(s) g_1 Z(s) \]
Use FACTS to Get the Impedance

- Obtain the impedance (transfer function) in a snapshot

\[ I_{\text{exc}}(s) \quad V_{\text{res}}(s) \]

\[ Z(s) = \frac{V_{\text{res}}(s)}{I_{\text{exc}}(s)} = \frac{Z_0}{1 + \frac{s}{s_p l}} \]

- For dc, open the capacitor

\[ Z_0 = R_1 \parallel R_{ac} \]

Response \[ \Omega \]

Excitation
No Algebra to Get the Result!

At the zero frequency, the response disappears

\[ r_c + \frac{1}{sC_{out}} = \frac{sr_C C_{out} + 1}{sC_{out}} \]
\[ sr_C C_{out} + 1 = 0 \]
\[ \omega_z = \frac{1}{r_C C_{out}} \]

Remove the excitation and look at the resistance driving \( C_{out} \)

\[ Z(s) = R_1 \parallel R_{ac} \frac{1 + sr_C C_{out}}{1 + sC_{out} (r_C + R_1 \parallel R_{ac})} \]
\[ \tau = \left[ r_C + \left( R_1 \parallel R_{ac} \right) \right] C_{out} \]
Final Expression

- Associate the impedance expression with $g_1$

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \frac{1 + \frac{s}{\omega_{z_1}}}{1 + \frac{s}{\omega_{p_1}}}$$

$$H_0 = \frac{V_{in}^2 V_c L}{T_{sw} (V_{out} - V_{in}) (S_c L + R_1 V_{in})^2} (R_{ac} \parallel R_1)$$

$$\omega_{z_1} = \frac{1}{r_C C_{out}}$$

$$\omega_{p_1} = \frac{1}{C_{out} (r_C + R_1 \parallel R_{ac})}$$

- However, we want the control-to-output current expression

$$\frac{V_{sense}(s)}{V_c(s)} = \frac{R_{sense}}{R_{sense} + r_{LEDs}} \frac{H_0 (1 + \frac{s}{\omega_{z_1}})}{1 + \frac{s}{\omega_{p_1}}}$$
Checking our Model Response

- Ac simulation of the current source-based approach

**parameters**

- $R_i = 0.25$
- $v_c = 0.4$
- $S_e = 100k$
- $F_{sw} = 1 Meg$
- $L = 3.3u$
- $T_{sw} = 1/F_{sw}$
- $V_{in} = 12$
- $V_{out} = 32.85$
- $k_1 = V_{in}^2 v_c L$
- $k_2 = T_{sw} (V_{out} - V_{in}) (S_e L + R_i V_{in})^2$
- $R_1 = 2 T_{sw} (V_{in} - V_{out})^2 (S_e L + R_i V_{in})^2 / (V_c^2 (V_{in}^2) L)$

![Circuit Diagram](image)
Simplified Approach vs PWM Switch

- The comprehensive model with the PWM switch

Parameters:
- $R_i = -0.25$
- $v_c = 0.4$
Final Results

- The RHPZ effect is not modeled in the simplified approach
Real Measurement vs Model

- Deviation occurs, as expected, because of the RHPZ

3.3uH
Control to Output (R29 current sense)

- Model
- Prototype

C. Basso, A. Laprade, "Simplified Analysis of a DCM Boost Converter Driving an LED String", www.how2power.com
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Why Bordeline Operation?

- More converters are using variable-frequency operation
- This is known as Quasi-Square Wave Resonant mode: QR
  - Valley switching ensures extremely low capacitive losses
  - DCM operation saves losses in the secondary-side diode
  - Easier synchronous rectification
  - The Right Half-Plane Zero is pushed to high frequencies

![Diagram showing smooth signals, less noise, and low CV^2 losses](image)
What is the Principle of Operation?

- The drain-source signal is made of peaks and valleys
- A valley presence means:
  - The drain is at a minimum level, capacitors are naturally discharged
  - The converter is operating in the discontinuous conduction mode

 BCM = Borderline or Boundary Conduction Mode

Flyback structure
A QR Circuit Does not Need a Clock

- The system is a self-oscillating current-mode converter

![QR Circuit Diagram]

$$V_{in}$$

$$V_{out}$$

2.5 V

50 mV

$$L$$

$$C$$

$$R$$

$$v_c(t)$$

$$v_{sense}(t)$$

$$v_t$$

$$d(t)$$

$$S$$

$$Q$$

$$R_i$$
A Winding is Used to Detect Core Reset

- When the flux returns to zero, the aux. voltage drops
- Discontinuous mode is always maintained

\[ v_{aux}(t) = -N \frac{d\varphi(t)}{dt} \]

\[ v_{aux}(t) = \frac{d\varphi(t)}{dt} \]

\[ V_{DS}(t) \]

\[ \varphi = 0 \]
Test the Large-Signal Model Response

- Insert the PWM BCM CM model in the boost converter

![Circuit Diagram]

- **Parameters**
  - $L = 22\, \mu\text{H}$
  - $R_i = -0.1$
SPICE Gives Us the Response

- SPICE linearizes the model for us around the bias point.
Derive a Small-Signal Model

A Quasi Resonant model is built with the PWM switch model

\[
D = \frac{V_c}{R_i} \frac{L}{V_{ac} T_{sw}} \quad T_{sw} = \frac{V_c L}{R_i} \left( \frac{1}{V_{ac}} + \frac{1}{V_{cp}} \right)
\]

\[
I_a = \frac{V_c}{R_i} \frac{L}{V_{ac} T_{sw}} \quad I_c = \frac{I_c}{V_{ac}} \left( \frac{1}{V_{ac}} + \frac{1}{V_{cp}} \right)
\]

\[
I_c = \frac{V_c}{2R_i} \quad 1 \text{ variable}
\]

These are large-signal equations that need linearization

\[
I_c = f(V_c) \quad \Rightarrow \quad \hat{i}_c = \frac{\partial I_c(V_c)}{\partial V_c} \hat{v}_c \quad \hat{i}_c = \hat{v}_c \left( \frac{1}{2R_i} \right) = \hat{v}_c k_c \quad k_c = \frac{1}{2R_i}
\]

\[
\hat{i}_a = \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial V_{cp}} \bigg|_{I_c, V_{ac}} \hat{v}_{cp} + \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial I_c} \bigg|_{V_{ap}, V_{ac}} \hat{i}_c + \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial V_{ac}} \bigg|_{I_c, V_{cp}} \hat{v}_{ac}
\]
Large to Small-Signal

- Final steps before the small-signal model

\[
\hat{i}_a = \hat{v}_{cp} \frac{I_{c0} V_{ac0}}{(V_{ac0} + V_{cp0})^2} + \frac{V_{cp0}}{V_{cp0} + V_{ac0}} \hat{i}_c - \hat{v}_{ac} \frac{V_{cp0} I_{c0}}{(V_{ac0} + V_{cp0})^2} \\

k_{cp} = \frac{I_{c0} V_{ac0}}{(V_{ac0} + V_{cp0})^2} \hspace{1cm} k_{ic} = \frac{V_{cp0}}{V_{cp0} + V_{ac0}} \hspace{1cm} k_{ac} = \frac{V_{cp0} I_{c0}}{(V_{ac0} + V_{cp0})^2}
\]

- A small-signal model can now be assembled
Always Check the Small-Signal Response

- Always verify if coefficients are well derived!

parameters

- L = 22u
- Ri = -0.1
- Vin = 10
- Vout = 15.6
- M = Vout / Vin
- Ic = 2.5
- Vac = -10
- Vap = -Vout
- Vcp = Vin - Vout
- kcp = Ic * Vac / (Vac + Vcp)^2
- kic = Vcp / (Vcp + Vac)
- kac = Vcp * Ic / (Vac + Vcp)^2
- kc = 1 / (2 * Ri)

Vin = 10
Vout = 15.6
M = Vout / Vin
Ic = 2.5
Vac = -10
Vap = -Vout
Vcp = Vin - Vout
kcp = Ic * Vac / (Vac + Vcp)^2
kic = Vcp / (Vcp + Vac)
kac = Vcp * Ic / (Vac + Vcp)^2
kc = 1 / (2 * Ri)
A Simple Intermediate Sanity Check

- Same response as with the non-linear model: good to go
The Model at Work in the BCM Boost

- Replace the switch/diode in the boost configuration

- In the original model, $I_c$ leaves node c. It enters it in a boost.
Identify Static Parameters in the Coefficients

- Depending on configurations, update variables in coefficients

\[ V_{cp0} = V_{in} - V_{out} \]
\[ I_{c0} = -\frac{I_{out} V_{out}}{V_{in}} \]
\[ V_{ac0} = -V_{in} \]

\[ k_{cp} = \frac{I_{out} V_{out}}{V_{in}} \frac{V_{in}}{(-V_{in} + V_{in} - V_{out})^2} \]
\[ k_{ac} = -\frac{I_{out} V_{out}}{V_{in}} \frac{(V_{in} - V_{out})}{(-V_{in} + V_{in} - V_{out})^2} \]
\[ k_{ic} = \frac{V_{in} - V_{out}}{V_{in} - V_{out} - V_{in}} \]

\[ k_{cp} = \frac{1}{R} \quad k_{ac} = \frac{M}{R} - \frac{1}{R} \quad k_{ic} = 1 - \frac{1}{M} \quad k_{c} = \frac{1}{2R_i} \]
First Step is Dc Gain

- Open the output capacitor, short the inductor

- Write output voltage expression

\[ I_{out}(s) = V_c(s)k_c + V_{cp}(s)k_{cp} - I_c(s)k_{ic} - V_{ac}(s)k_{ac} \]

\[ \rightarrow I_{out}(s) = V_c(s)k_c + (V_{(c)}(s) - V_{(p)}(s))k_{cp} - V_c(s)k_c k_{ic} - (V_{(a)}(s) - V_{(c)}(s))k_{ac} \]
Dc Gain Derivation

- There is no contribution from the source, $V_{in}(s) = 0$

$$I_{out}(s) = V_c(s) k_c - V_{out}(s) k_{cp} - V_c(s) k_c k_{ic} = V_c(s) k_c (1 - k_{ic}) - V_{out}(s) k_{cp}$$

$$I_{out}(s) = \frac{V_{out}(s)}{R} \quad \Rightarrow \quad V_c(s) k_c (1 - k_{ic}) = V_{out}(s) \left( k_{cp} + \frac{1}{R} \right)$$

$$\frac{V_{out}(s)}{V_c(s)} = \frac{k_c (1 - k_{ic})}{k_{cp} + \frac{1}{R}} \quad \Rightarrow \quad H_0 = \frac{R}{4MR_i}$$

- We have the first term of our transfer function

$$\frac{V_{out}(s)}{V_c(s)} = H_0 \left( 1 + \frac{s}{s_{p_1}} \right) \left( 1 + \frac{s}{s_{z_1}} \right)$$
Deriving the Zero Position

For the zero, the stimulus does not reach the output

- current in the resistance is 0, node p voltage is also 0

All the inductor ac current is absorbed before reaching R

\[ V_c(s)k_c + \left[ V_c(s) - V_p(s) \right]k_{cp} = V_c(s)k_c k_{ic} + \left[ V_a(s) - V_c(s) \right]k_{ac} \]
What is the Root to $N(s) = 0$?

- The voltage at node c depends on the inductance
  \[ V_c(s) = -V_L(s) = -I_c(s)sL = -V_c(s)k_c sL \]

- Substitute in the previous equations and simplify
  \[ V_c(s)k_c - V_c(s)k_c sL k_{cp} = V_c(s)k_i k_{ic} + V_c(s)k_c sL k_{ac} \]

- Solve for $s$, this is the zero position
  \[ 1 - k_{ic} = sL(k_{ac} + k_{cp}) \quad \rightarrow \quad s_{z_1} = \frac{1 - k_{ic}}{L(k_{ac} + k_{cp})} \quad \text{Substitute} \quad k_{ic}, k_{ac}, k_{cp} \]

- The root is positive, this is a Right Half Plane Zero
  \[ s_{z_1} = \frac{R}{LM^2} \]
For the Pole, Reduce Excitation to 0

- Excitation is \( V_c \): all current sources \( f(V_c) \) are open

\[ V_c (s) = 0 \]

- Look for the resistance “seen” by the capacitor \( C \)
- The source \( V_{cp}k_{cp} \) can be reworked

\[
V_{cp}(s)k_{cp} = -V_p(s)k_{cp} = -\frac{V_{out}(s)}{R} \rightarrow \text{Replace by a resistance } R
\]

\[ \hat{v}_c = 0 \]
A Really Simple Circuit

- Difficult to beat in terms of problem solving

- The pole due to the capacitor comes immediately

\[ R_\text{?} = R \parallel R = \frac{R}{2} \rightarrow \tau = \frac{R}{2}C \rightarrow s_{p_1} = \frac{2}{RC} \]
The Complete Expression is Ready

- With a few steps, we have our transfer function

\[
\frac{V_{\text{out}}(s)}{V_c(s)} = H_0 \frac{1 - \frac{s}{\omega_{z_1}}}{1 + \frac{s}{\omega_{p_1}}}
\]

\[
H_0 = \frac{R}{4MR_i}, \quad \omega_{p_1} = \frac{2}{RC}, \quad \omega_{z_1} = \frac{R}{LM^2}
\]

- We can easily add the capacitor ESR contribution

\[
r_C + \frac{1}{sC} = \frac{sr_C C + 1}{sC}
\]

\[
sr_C C + 1 = 0
\]

\[
\omega_{z_2} = \frac{1}{r_C C}
\]

\[
\frac{V_{\text{out}}(s)}{V_c(s)} = H_0 \left( 1 - \frac{s}{\omega_{z_1}} \right) \left( 1 + \frac{s}{\omega_{z_2}} \right)
\]

\[
1 + \frac{s}{\omega_{p_1}}
\]
Time to Confront Mathcad® with SPICE!

- If equations are correct, curves must superimpose perfectly

- The BCM boost response in CM is first-order with RHPZ
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Active Clamp Forward in Voltage Mode

- An Active Clamp Forward (ACF) is a forward converter...

- …featuring a controlled-upper-side switch for ZVS operations
Control Strategy

- A deadtime is inserted to let $V_{DS}$ swing towards grounds

- Quasi-ZVS can be implemented on the drain voltage
Active Clamp Forward in Voltage Mode

- Energy is stored in the magnetizing inductance at turn on

\[ L_i(t) \]

\[ C_{\text{mag}} L_i(t) \]

- We need to offer a path at turn off to demagnetize the core

\[ i_{\text{mag}}(t) + N_i L_i(t) \]
All Parasitic Capacitors are Charged

- At turn off, the current charges the lump capacitor

- The drain voltage increases until it touches $V_{clamp}$
Magnetizing Current Resonates

- The upper-side MOSFET switches on with delay: ZVS

\[ i_{mag}(t) \]

Magnetizing current decreases to 0 then reverses
Deadtime Gives Quasi-ZVS Operation

- The deadtime lets magnetizing current reach $-I_{mag,peak}$

At this moment, $Q_2$ opens and current discharges $C_{lump}$
Full ZVS at Nominal Power is Difficult

- True ZVS is difficult to reach, quasi-ZVS is usually obtained

\[ v_{DS}(t) \]

\[ i_{mag}(t) \]

\[ V_{clamp} \]

\[ V_{\text{clamp}} \]

\[ 0 \]

\[ 0 \]

\[ \text{max} \]

\[ -I_{\text{mag, peak}} \]

\[ Q_2 \text{ opens} \]

\[ Q_1 \text{ closes} \]

True ZVS

\[ \frac{1}{2} L_{\text{mag}} \left( I_{\text{mag, peak}} - NI_L \right)^2 > \frac{1}{2} C_{\text{lump}} V_{\text{in}}^2 \]
Building an Ac Model

- A model can be built following different methods
  - Write large-signal equations of voltages and currents
  - Assemble sources to build a large-signal model
  - Linearize expressions and derive transfer functions
    - Reveal the presence of the PWM Switch model
    - Implant its already-available small-signal model
    - Solve for the transfer function expression

- Both approaches have pros and cons
- Going along both paths helps to cross-check results
Identify PWM Switches Pair

- A pair of switches appear in primary and secondary sides
Primary-Side PWM Switch Model

- Identify currents and voltages during transitions

\[ \langle v_{DS-on}(t) \rangle_{DT_{sw}} = \left( I_{mag} + NI_{out} \right) r_{on1} \]

During the on-time \( DT_{sw} \)

\[ \langle v_{DS-off}(t) \rangle_{(1-D)T_{sw}} = V_{in} + V_C + r_{on2} I_{mag} \]

During the off-time \((1-D)T_{sw}\)
The First PWM Switch Model is Here

- Connect the PWM Switch the right way, check polarities

Connect

\[ V_{in} \]

\[ r_{on2} I_c (1 - D) I_c \]

\[ C_{clp} \]

\[ L_{mag} \]

\[ V_{loss} \]

\[ p \]

\[ c \]

\[ a \]

\[ -V_{ap} \]

\[ -DI_c \]

\[ V_{ap} (D - 1) \]

\[ V_{pa} (1 - D) \]

\[ p \]

\[ c \]

\[ a \]

\[ V_{clamp} \]

\[ \langle v_{DS} (t) \rangle_{T_{sw}} \]

\[ L_{mag} \]

\[ I_c = I_{mag} \]

\[ V_{pa} (1 - D) \]
Secondary-Side PWM Switch Model

- The forward converter is a buck-derived topology

\[ V_{\text{loss}} = \left( I_{\text{mag}} + NI_{\text{out}} \right) r_{\text{on1}} \]

Primary-side loss

\[ V_{\text{loss}} = \left( I_{\text{mag}} + \frac{I_{p}}{D} \right) r_{\text{on1}} \]
Check the Dual-PWM Switch Response

- Compare ac response with that of the large-signal version

- Check first if dc operating points are similar
- Use this fixture to also run transient simulations
Large-Signal Equations with SPICE

- Non-linear equations are linearized by SPICE

Non-linear equations are linearized by SPICE

1. \( V(D) \cdot N \cdot (V\text{in}) - (V\text{in}) \cdot \text{Ron1} (1 - V(D)) \cdot I(VLP) + I(VIL) \cdot N \) - \( Vf \)

2. \( (V\text{in}) + V\text{clamp} + \text{Ron2} (1 - V(D)) \cdot I(VLP) + V(D) \cdot \text{Ron1} (1 - V(D)) \cdot I(VLP) + I(V5) \cdot V(D) + I(V) 

Parameters:
- \( N = 1/6 \)
- \( \text{Ron1} = 60 \text{m} \)
- \( \text{Ron2} = 60 \text{m} \)
- \( Vf = 0.5 \)

Proposed by Dr. José Capilla, ON Semi, November 2012
Curves Perfectly Superimpose

\[ \frac{V_{\text{out}}(f)}{D(f)} \]

Control-to-output transfer function magnitude

\[ \frac{V_{\text{clamp}}(f)}{D(f)} \]

Control-to-clamp-voltage transfer function

\[ \frac{I_{\text{in}}(f)}{D(f)} \]

Input current response

\[ \angle \frac{V_{\text{out}}(f)}{D(f)} \]

Control-to-output transfer function argument
Compare Transient Responses

- Cycle-by-cycle results are similar to that of average model

![Graphs showing transient responses with plots for $v_{out}(t)$, $i_{mag}(t)$, and $v_{clamp}(t)$]
Simulate with Small-Signal Sources

- Plug the small-signal models of the PWM Switch in

- Check if ac response is ok before proceeding!
- Simplify circuitry to concentrate on control-to-output only
Same Response Between Fixtures

\[
\left| \frac{V_{out}(f)}{D(f)} \right|
\]

\[
\angle \frac{V_{out}(f)}{D(f)}
\]
Make the Circuit Look Friendlier

- Re-arrange sources to simplify the electrical circuit

- Run a sanity check further to any simplification

---

**Mag. current generator**

- **B2 Voltage**
  - V(1,2) * \( r(2) \) * (V(LP))

- **B4 Voltage**
  - V(B,4) * (V(6)) * (V(dac))

- **B9 Current**
  - I(VLP) * N

- **B3 Current**
  - I(VLP) * V(dac) / N

**Isolated buck converter**

- **B8 Voltage**
  - \( \{\text{Vf}\} \) * \( \{\text{Iout}\} \) * V(dac)
  - \( \{\text{ron1}\} \) * \( \{\text{ron2}\} \) * (I(VLP)) / (\( \{\text{Vout}\} \) / (\( \{\text{Iout}\} \) * V(LP))) / (\( \{\text{D}\} \))

- **B10 Voltage**
  - \( \{\text{Vf}\} \) * (V(4,0)) * V(dac) / (\( \{\text{D}\} \))

- **B6 Voltage**
  - \( \{\text{Vf}\} \) * (V(4,0)) * V(dac) / (\( \{\text{D}\} \))

**parameters**

- \( N = 1/6 \)
- \( \text{ron1} = 60 \Omega \)
- \( \text{ron2} = 60 \Omega \)
- \( \text{Vf} = 0.5 \)
- \( \text{Iout} = 33 \)
- \( \text{D} = 478.279 \)

\[ \text{dac} = \hat{d} \]

\[ D = D_0 \]
Circuit is Looking Simpler Now

- Look for ways to get simpler equations, final arrangement

- Final ac check is mandatory!
Simpler Circuit Does Not Distort Response

We are good to start analyzing the equivalent circuit
Start with the Clamp Circuitry

For the dc transfer function, open caps and short inductors

parameters

\[ V_{clp} = V(c)D = V_{clamp}D \]

\[ V_{clamp} = V_{in} + V_{clp} \]

\[ V_{clp} = \left( V_{in} + V_{clp} \right)D \]

\[ V_{clp} = V_{in} \frac{D}{1 - D} = 48 \frac{0.478}{1 - 0.478} \approx 44 \text{ V} \]

\[ V_{clamp} = V_{in} + V_{clp} = 92 \text{ V} \]

This is a buck-boost dc transfer function
We Need the Magnetizing Current

- Use KVL and KCL to get the mag. current expression

\[ V_{L_{mag}} = V_{(c)} - V_{(c)} \left( \hat{d} + D_0 \right) + D_0 r_{on1}\hat{i}_{mag} \]

The secondary side contribution has been purposely neglected, \( I_{out} \) and \( \hat{i}_{out} \)
Collect Terms and (Carefully) Simplify

- Use KVL and KCL to get the mag. current expression
  \[ V_{L_{mag}} = V(2) - V_{in} \quad \text{with} \quad V(2) = V(C) - V(C)(D_0 + \hat{d}) + D_0 r_{on1}\hat{i}_{mag} \]

The voltage at node (c) has a dc and an ac component

\[ V_{L_{mag}} + \hat{v}_{L_{mag}} = V(C) + \hat{v}(C) - (V(C) + \hat{v}(C))\left(D_0 + \hat{d}\right) + D_0 r_{on1}\hat{i}_{mag} - V_{in} - \hat{v}_{in} \]

\[ V_{L_{mag}} + \hat{v}_{L_{mag}} = V(C) + \hat{v}(C) - V(C)\hat{d} - V(C)D_0 - \hat{v}(C)\hat{d} - \hat{v}(C)D_0 + D_0 r_{on1}\hat{i}_{mag} - V_{in} - \hat{v}_{in} \approx 0 \]

- Sort out an ac and a dc equation

  **dc** \[ V_{L_{mag}} = V(C) - V(C)D_0 - V_{in} \quad \text{and} \quad \langle V_{L_{mag}} \rangle_{T_{pw}} = 0 \quad \Rightarrow \quad V_{in} = V(C)(1 - D_0) \quad \Rightarrow \quad V_{clamp} = \frac{V_{in}}{(1 - D_0)} \]

  **ac** \[ \hat{v}_{L_{mag}} = \hat{v}(C)(1 - D_0) - V_{clamp}\hat{d} + D_0 r_{on1}\hat{i}_{mag} \]
Express the Magnetizing Current $i_{\text{mag}}$

- The clamp capacitor ac voltage depends on $i_{\text{mag}}$

$$\hat{v}_c = i_{\text{mag}} (1 - D_0) \left( \frac{1}{sC_{clp}} \right) + i_{\text{mag}} r_{on2} = i_{\text{mag}} \left[ (1 - D_0) \left( \frac{1}{sC_{clp}} \right) + r_{on2} \right]$$

Substitute

$$i_{\text{mag}} = -\frac{\hat{v}_{L_{mag}}}{sL_{mag}} = -\frac{\hat{v}_c (1 - D_0) - V_{clamp} \hat{d} + D_0 r_{on1} \hat{i}_{\text{mag}}}{sL_{mag}}$$

Solve for $i_{\text{mag}}$

$$I_{\text{mag}}(s) = D(s)V_{clamp} \frac{sC_{clp}}{D_0^2 - 2D_0 + 1 + sC_{clp} (r_{on2} + D_0 r_{on1} - D_0 r_{on2}) + s^2 L_{mag} C_{clp}}$$
Identify Second-Order Coefficients

- Identify terms with a second-order polynomial form

\[
\frac{I_{\text{mag}}(s)}{D(s)} = \frac{V_{\text{clamp}}}{(1 - D_0)^2} \frac{sC_{\text{clp}}}{1 + sC_{\text{clp}}} \left[ \frac{r_{on2}(1 - D_0) + D_0r_{on1}}{(1 - D_0)^2} \right] + s^2 \frac{L_{\text{mag}}C_{\text{clp}}}{(1 - D_0)^2}
\]

Develop and rearrange:

\[
\frac{I_{\text{mag}}(s)}{D(s)} = M_0 \frac{sC_{\text{clp}}}{1 + \frac{s}{\omega_0M} \left( \frac{s}{\omega_0M} \right)^2}
\]

\[
M_0 = \frac{V_{\text{clamp}}}{(1 - D_0)^2} = \frac{V_{\text{in}}}{(1 - D_0)^3} \quad \omega_0M = \frac{1 - D_0}{\sqrt{L_{\text{mag}}C_{\text{clp}}}}
\]

\[
Q_M = \sqrt{\frac{L_{\text{mag}}}{C_{\text{clp}}} \frac{1 - D_0}{r_{on2}(1 - D_0) + D_0r_{on1}}}
\]
Re-Write the Expression Nicely

A tuned network offers the following transfer function

\[
H(s) = A_0 \frac{1}{1 + \left(\frac{\omega_0}{s} + \frac{s}{\omega_0}\right)Q}
\]

\[
I_{mag}(s) = \frac{M_0 s C_{clp}}{D(s)} \left[1 + \frac{s}{\omega_0 Q M} + \left(\frac{s}{\omega_0 M}\right)^2\right]
\]

\[
A_0 = \frac{V_{clamp}}{r_{on2} (1 - D_0) + D_0 r_{on1}} \quad \Rightarrow \quad |A_0(\omega_0M)| = 67.713 \text{ dB}
\]
Time for an ac Check

- SPICE can obtain the ac response in a snap-shot

Parameters:
- $N = \frac{1}{6}$
- $\text{ron1} = 60\,\text{m}$
- $\text{ron2} = 60\,\text{m}$
- $V_f = 0.5$
- $I_{out} = 33$
- $V_{in} = 48$
- $D = 478.279\,\text{m}$

SPICE can obtain the ac response in a snap-shot.
Curves Perfectly Superimpose

- It is important to obtain similar plots, otherwise: error!

Max SPICE is 63.710 dB
Max Mathcad® is 63.713 dB

"Can do!"
Run Another Round of Rearrangement

Now concentrate on the buck output stage only

\[ V(7) = NV_{in} - r_{on1}N^2 \left( \frac{I_{mag}}{N} + I_{out} \right) \]
Further Simplification is Necessary

Extract the ac current from the equation, \( \langle i_{\text{mag}}(t) \rangle_{T_{sw}} = 0 \)

\[
V_{(1)} = NV_{\text{in}} - r_{on1} N^2 \left( \hat{i}_{\text{mag}} + (I_{\text{out}} + \hat{i}_{\text{out}}) \right) \quad \rightarrow \quad V_{(1)} = \left[ NV_{\text{in}} - r_{on1} N^2 \left( \hat{i}_{\text{mag}} + (I_{\text{out}} + \hat{i}_{\text{out}}) \right) \right] (D + \hat{d})
\]

Develop \( V_{(1)} \) keep ac terms only

\[
DNV_{\text{in}} + NV_{\text{in}} \hat{d} - DN \hat{i}_{\text{mag}} r_{on1} - N \hat{d} \hat{i}_{\text{mag}} r_{on1} - DI_{\text{out}} N^2 r_{on1} - DN^2 \hat{i}_{\text{out}} r_{on1} - I_{\text{out}} N^2 \hat{d} r_{on1} - N^2 \hat{d} \hat{i}_{\text{out}} r_{on1}
\]

dc \quad \approx 0 \quad dc \quad \approx 0

Rearranging the result, we obtain:

\[
NV_{\text{in}} \hat{d} - DN \hat{i}_{\text{mag}} r_{on1} - DI_{\text{out}} N^2 r_{on1} - DN^2 \hat{i}_{\text{out}} r_{on1} - I_{\text{out}} N^2 \hat{d} r_{on1}
\]

\( r_{on1} \ll 1 \)

\[
\hat{d} \left( NV_{\text{in}} - I_{\text{out}} N^2 \hat{d} r_{on1} \right) - DN \hat{i}_{\text{mag}} r_{on1} - DN^2 \hat{i}_{\text{out}} r_{on1}
\]

\( \hat{d} \ll 1 \)

Neglecting small terms, we finally obtain:

\[
\hat{v}_{(1)} = \hat{d} NV_{\text{in}} - DN \hat{i}_{\text{mag}} r_{on1}
\]
Add the Second-Order Response of \( LC \) Filter

The magnetizing current definition is:

\[
I_{mag}(s) = M_0 \frac{sC_{clp}}{D(s)} = M(s)
\]

\[
D(s) = 1 + \frac{s}{\omega_{0M} Q_M} + \left( \frac{s}{\omega_{0M}} \right)^2
\]

The source is filtered by the 2\(^{nd}\)-order \( LC \) filter:

\[
F(s) = F_0 \frac{1 + \frac{s}{s_{ZF}}}{1 + \frac{s}{\omega_{0F} Q_F} + \left( \frac{s}{\omega_{0F}} \right)^2}
\]

\[
F_0 = \frac{R_{load}}{r_L + r_{L load}} \quad \omega_{ZF} = \frac{1}{r_C C_{out}}
\]

\[
Q_F = \frac{L_{out} C_{out} \omega_{0F} (r_C + R_{load})}{L_{out} + C_{out} (r_L r_C + R_{load} (r_L + r_C))}
\]
We Have the Final Transfer Function

- The transfer function reveals the mag. current contribution

\[
\frac{V_{\text{out}}(s)}{D(s)} = F(s)(N V_{\text{in}} - D_0 N r_{on1} M(s)) \rightarrow \text{YES!}
\]

- The mag. current substracts and explains the notch

\[
\frac{V_{\text{out}}(s)}{D(s)} = F_0 \left( 1 + \frac{s}{s_{ZF}} \right)^N \left( V_{\text{in}} - D_0 r_{on1} M_0 \right) \left( \frac{s C_{clp}}{1 + \frac{s}{\omega_{0M} Q_{M}} + \left( \frac{s}{\omega_{0M}} \right)^2} \right)
\]

- This is the control-to-output transfer function of the ACF
Final Sanity Check - Magnitude

- Compare the analytical ac response with that of SPICE

- Magnitude curves superimpose perfectly

\[ 20 \cdot \log \left( \frac{\left| H_1(i \cdot 2\pi \cdot f_k) \right|}{V} \right) \text{, } 10 \]

\[ \frac{V_{\text{out}}(f)}{D(f)} \]
Final Sanity Check - Phase

- Compare the analytical ac response with that of SPICE

\[ \left( \text{arg}(H_1(i \cdot 2\pi f_k)) \cdot \frac{180}{\pi} \right) \]

- Argument curves are in excellent agreement too
The Bench is the Final Referee

- Build a hardware using simple gates arrangements

- This active clamp forward converter delivers 5 V/5 A
The Prototype Hardware

- The circuit is assembled using available components

- Make sure caps. are well characterized before soldering

With the kind help of Yann Vaquette, Application engineer
Typical Prototype Waveforms

$V_{outA}(t)$

$V_{outM}(t)$

$V_{DS}(t)$

N-channel
Quasi-ZVS is Ensured on the Drain

\[ V_{in} = 51 \text{ V} \]

100 V

28 V

140 ns

N-channel
ZVS is Also Ensured on Clamp Switch

\[
v_{outA}(t) \quad v_{outM}(t) \quad 240 \text{ ns} \quad \text{Body diode} \quad v_{DS}(t)
\]

\[
P\text{-channel} \quad -100 \text{ V}
\]
Current in the Clamping Network

\[ v_{outA}(t) \]

\[ v_{outM}(t) \]

\[ i_{C_{clamp}}(t) \]
We have used the following SPICE simulation fixture:

- Parameters:
  - $N = 0.25$
  - $L_{mag} = 580\mu H$
  - $r_{on1} = 0.9$
  - $r_{on2} = 0.6$
  - $V_f = 0.65$

- Check dc points versus hardware values: ok
Magnitude Response

- There is a slight shift but overall agreement is good

- cap. ESR is often the offender in loop measurement
Phase Response

- Good agreement between curves, especially peaking

The notch $Q$ depends on resistive elements $r_{DS(on)}$ etc.
Comparison with Simplis Response

- Excellent agreement between curves, almost no shift
Comparison with Simplis Response

- Despite a slightly lower peaking, phase agreement is ok

\[ \angle H(f) \]
Simulation also Confirms Damping!

- Inserting a 10-Ω resistance damps the notch nicely
A Practical Case

- The model helped to stabilize this 3.3-V/30-A converter

\[ f_c = 31.9 \text{ kHz} \]
\[ \varphi_m = 62.9^\circ \]
Transient Response Test

- Step-load transient response confirms stability margins

\[ v_{OUT}(t) \]

Overshoot = 42.2 mV

Undershoot = 48.2 mV

\[ V_{IN} = 36 \text{ V}, 15 \text{ A to 22.5 A} - \text{Slew rate 1 A/\mu s} \]
Conclusion

- The PWM switch model is an essential tool for modeling
- We have seen how to derive it in different operating modes
- Small-signal modeling using the PWM switch is simple and fast
- When modeling converters, always proceed step by step
- Always perform intermediate sanity checks (SPICE, Mathcad\textsuperscript{®}…)
- Analytical analysis does not shield you against lab. experiments
- Analysis, simulation and bench: the path to success!

Merci !
Thank you!
Xiè-xie!