Simulation and Analysis Applied to the Design of Buck Topologies

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Chris Basso APEC Seminars
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Tenth anniversary!
Course Agenda

- The Buck Converter
- Control Schemes
- Introduction to Modeling
- The PWM Switch in Current-Mode Control
- Introduction to the Fast Analytical Techniques
- The Four Transfer Functions in CCM Current-Mode Control
- Compensation Strategy
- Prototype Measurements
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Input Voltage Reduction: Linear or Switching?

- How do you step-down the input source?
  - A linear regulator
    - Poor efficiency at high input-output differentials
    - Constrained to low input voltages
    - Low-noise linear operation
  - A switching regulator
    - Noisy operation, requires filtering
    - Built with energy-storing components
    - Excellent efficiency
    - Works across a wide input voltage range
Principles of Operation in Voltage-Mode Control

- The buck converter can be operated in voltage-mode control
  - Control-to-output transfer function changes between operating modes
- No need to sense the inductor current

- Inherently-low open-loop output impedance
- Mediocre input line rejection

* Beside over-current protection purposes
The buck converter can be operated in current-mode control

1st-order response in low frequencies whether CCM or DCM operation

- Unstable for duty ratios approaching 50% — CCM needs slope compensation

- Mediocre open-loop output impedance

- Inherent good input line rejection
The Two Operating Phases of the Buck Converter

Neglecting drops, the common $SW-D$ node swings between $V_{in}$ and 0 V.

During $DT_{sw}$

$$I_{peak} = \frac{V_{in} - V_{out}}{L_1}$$

During $(1 - D)T_{sw}$

$$I_{valley} = -\frac{V_{out}}{L_1}$$

Low-impedance square generator

2nd-order system
A Third State Exists when the Inductor Depletes

- Observing the inductor current to infer conduction mode

- CCM – continuous conduction mode

- BCM – borderline conduction mode

- DCM – discontinuous conduction mode

- Inductive current is in the high-side branch: how to sense?
Challenging Current Sensing for Current Mode

- Insert a resistance with $L$

  ![Diagram](image)

  - $i_L(t)$
  - $R_{sense}$
  - $L$
  - $v_{CS}(t)$
  - $v_{CS}(t)A_v$

  ✓ Simplest solution
  ✓ Can read $I_{out}$
  ❖ Efficiency suffers

- Use MOSFET $r_{DS(on)}$

  ![Diagram](image)

  - $i_D(t)r_{DS(on)}A_v$
  - $v_{DS}(t)$
  - $L$

  ✓ No extra sensing element
  ✓ Can be integrated
  ❖ Temperature-dependent values
  ❖ $r_{DS(on)}$ not tested in production
Dc Resistance Inductor Current Sensing

- Integrate inductor voltage

\[ V_{CS}(s) \approx I_L(s) r_L \frac{1 + s \frac{L}{r_L}}{1 + sRC} \]
\[ V_{CS}(s) = I_L(s) r_L \frac{1 + s\tau_1}{1 + s\tau_2} \]

- Time constant mismatch
  - inductance varies with A and °C
  - beware of cheap material

- Most efficient solution
- Low cost

Sense Transformer for High-Current Applications

- The transformer references the measured variable to ground
- Turns ratio $N$ emulates any sense resistance value

![Diagram of sense transformer](image)

\[ R_{\text{sense}} = \frac{V_{\text{sense,max}}}{I_{D,\text{max}}} = \frac{0.5 \text{ V}}{30 \text{ A}} = 16.6 \text{ m}\Omega \]

\[ R_{\text{burden}} = \frac{V_{\text{sense,max}}}{I_{D,\text{max}}} N = \frac{0.5 \text{ V}}{30 \text{ A}} \times 100 = 1.66 \Omega \]

- Transformer reset is necessary (e.g. with a Zener diode)
- Can only sense ac currents
Parasitic Terms Affect Conversion Ratio

- Apply volt-second balance law*: \( \langle v_L(t) \rangle_{T_{sw}} = 0 \)

Inductor flux balance implies:
\( \langle v_L(t) \rangle_{t_{on}} - \langle v_L(t) \rangle_{t_{off}} = 0 \)

\[
\frac{V_{out}}{V_{in}} = M = \left[ \frac{1}{1 + \frac{r_{DS(on)}}{R} D + \frac{r_L}{R} + \frac{V_f}{V_{out}} D'} \right]
\]

* At steady-state
Analyzing the Conversion Ratio Equation

- Depending on the duty ratio, optimize one of the paths

\[
D \left[ \frac{1}{1 + \frac{r_{DS(on)}}{R} D + \frac{r_L}{R} + \frac{V_f}{V_{out}} D'} \right]
\]

Small duty ratio

Less burden on transistor

12 V to 1.8 V

Diode conduction

Synchronous rectification

Duty ratio scales losses

Large duty ratio

Less burden on diode

Reduce \( r_{DS(on)} \)

5 V to 3.3 V

Continuous conduction mode (CCM) is assumed

Public Information 2/2/2018
Synchronous Rectification

- Diode conduction losses depend on average and rms currents
  - As a 1st-order approximation, losses are insensitive to ripple current

$$P_d = V_{T0} I_{d,\text{avg}} + r_d I_{d,\text{rms}}^2 \approx V_f I_{d,\text{avg}}$$

- Replace the diode by a MOSFET and ripple current now matters

$$P_Q = I_{Q,rms}^2 r_{DS(on)} = (1 - D) \left( I_{out}^2 + \frac{\Delta I_L^2}{12} \right) r_{DS(on)}$$

Synchronous Rectification Body Diode Conduction

- Limit shoot-through currents by inserting a dead-time

- Body diode conduction can hamper efficiency

\[ P_Q = I_{Q,\text{rms}}^2 r_{DS(\text{on})} = (1 - D_{\text{eff}}) \left( I_{\text{out}}^2 + \frac{\Delta I_L}{12} \right) r_{DS(\text{on})} \]

\[ P_{\text{body}} = 2 \cdot \text{DT} \cdot F_{\text{sw}} I_{\text{out}} V_f \]

Minimize deadtime without shoot-through
Continuous Conduction in No-Load Conditions

- DCM operation is often associated with light- and no-load operation
- Synchronous rectification allows inductor current to swing below 0 A

![Graph showing inductor current over time](image)

- Single-switch DCM operation
- Light-load condition

With synchronous rectifiers, inductor current can swing below 0 A for different output currents: $I_{out} = 30$ A, $I_{out} = 20$ A, $I_{out} = 10$ A, and $I_{out} = 3$ A.
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Different Modulation Strategies

Trailing edge

- Clocked turn-on
- Fast turn-off
- Delayed turn-on

Leading edge

- Clocked turn-off
- Fast turn-on
- Delayed turn-off

Dual edge

- Fast turn-on
- Fast turn-off
Fixed Switching Frequency Operation

- The main switch turns on at the clock occurrence

Voltage mode

![Voltage mode diagram]

Current mode

![Current mode diagram]

- The duty ratio is directly controlled by $V_{err}$
- The peak current is controlled by $V_{err}$
- $D$ is indirectly controlled by $V_{err}$
The power stage is controlled via the duty ratio $D$

$$D = \frac{t_{on}}{T_{sw}} = \frac{3u}{10u} = 30\%$$

An artificial ramp is compared to the control voltage

$$v_{saw}(t) = V_p \frac{t}{T_{sw}}$$

$$v_{err}(t) = V_p d(t)$$

$$d(t) = \frac{v_{err}(t)}{V_p}$$
Modeling the PWM Block

- Its role is to convert a voltage $V_{err}$ into a duty ratio $D$

$$d(t) = \frac{v_{err}(t)}{V_p} \quad \text{average over } T_{sw} \quad D(V_{err}) = \frac{V_{err}}{V_p} \quad G_{PWM} = \frac{dD(V_{err})}{dV_{err}} = \frac{1}{V_p}$$

- It is a simple block inserted before the $D$ input of the model

![Diagram of PWM block]

**Modulator gain**

$$G_{PWM} = \frac{1}{V_p}$$

- Phase and magnitude are flat
- Frequency response of the naturally-sampled modulator is flat
- With a perfect comparator, $t_p = 0$
Current-Mode Operations

- The artificial ramp is replaced by the inductor current

In CCM with $D > 50\%$ subharmonic instabilities occur
Slope Compensation Cures Oscillations

- Injecting an external ramp damps the poles at $F_{sw}/2$

- Do not overcompensate
  - Current mode turns into voltage mode
The converters operate in borderline conduction mode.

- 200% inductor ripple current
- Load/line-dependent frequency
- Needs an extra winding over $L$

Reduced switching losses
Hysteretic Control – the Basic System

- The simplest and fastest switching converter

- Variable frequency operation
- Capacitor ESL affects stability in DCM

Hysteretic Control – Adding an Op-Amp

- You can add an error amplifier to improve dc regulation

- A compensation network is needed

- Many techniques to:
  stabilize $F_{SW}$, improve stability, reduce drop...

R. Redl, Ripple-Based Dc-Dc Converters, In-House Seminar, Toulouse 2010.
The two-phase extension requires a simple logic control.

- Bad dynamic current sharing (no sensing)
- Popular in dc-dc with OVP (motherboards)
On-time duration is fixed – restart is given by the valley current

- Can be subject to instabilities

\[ r_C C > \frac{t_{\text{off}}}{2} \]

Fixed-Off-Time Peak-Current Control - FOT

- Off-time duration is fixed – restart is given by the peak current

- Can be subject to instabilities

\[ r_C C > \frac{t_{on}}{2} \]

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Time-Discontinuous Switching Waveforms

A switching converter is made of linear elements

\[ V_{in} \quad r_{DS(on)} \quad L \quad r_L \quad C \quad \frac{V_{out}}{} \quad R_{load} \]

on time

\[ + \quad \frac{V_{out}}{} \quad V_{in} \quad r_d \quad L \quad r_L \quad C \quad \frac{R_{load}}{} \]

off time

The non-linearity or discontinuity is coming from transitions

Cannot differentiate

\[ v_{DRV}(t) \]

linear

Cannot differentiate

linear

\[ \text{on time} \quad \text{off time} \]

Singularity

\[ DT_{sw} \quad (1-D)T_{sw} \]

How do we get this?

Control-to-output transfer function

\[ \angle H(f) \quad |H(f)| \]
State Space Averaging Technique

- Despite linear networks, equation is discontinuous in time.
- Introduced in 76, SSA weights on and off expressions.

\[
\begin{align*}
\dot{x} &= \left[ A_1 D + A_2 (1 - D) \right] x(t) + \left[ B_1 D + B_2 (1 - D) \right] u(t) \\
&\quad \text{valid during (1-}D) T_{sw} \\
&\quad \text{valid during } DT_{sw}
\end{align*}
\]

- Singularity is gone (time continuous).
- Equation is nonlinear now.
- Linearize and feed the canonical model.
- Add a new element: restart from scratch!

The PWM Switch Model in Voltage Mode

- We know that non-linearity is brought by the switching cell

![Diagram of PWM switch in voltage mode]

- Why not linearize the cell alone?

![Simplified model of PWM switch]

The VM-PWM Switch Model is a Transformer

The PWM switch large-signal model is a dc "transformer"!

\[ I_a = DI_c \quad I_c = \frac{I_a}{D} \]

\[ V_{ap} = \frac{V_{cp}}{D} \quad V_{cp} = DV_{ap} \]

It can be plugged into any 2-switch CCM converter

Immediate results
Update the Schematic Diagram with the PWM Switch

- Like in a bipolar circuit, replace the switching cell...

- ...and solve a set of linear equations!
The Model fits all Converters Architectures

- The switching cell is everywhere: the model is invariant

- buck

- boost

- buck-boost

- Šuk

- flyback

- Ćuk

- zeta
Independent Switch Modeling: Direct Connections

- The switch and diode are individually modeled: easy substitution

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**B. Erickson, D. Maksimovic, Advances in Averaged Switch Modeling and Simulation, professional seminar, PESC, Charleston, 1999**
Peak-Current-Mode Control Models

- The control voltage $V_c$ adjusts the inductor peak current
- the duty ratio is indirectly controlled by $V_c$

- Direct duty ratio control
- Indirect duty ratio control

Power stage

$V_{in} \rightarrow V_{out}$

$D$  

$V_{err}$ stage

PWM stage

$V_{in} \rightarrow V_{out}$

$D$

$S_{on}$ $I_p$ $\Delta I_L$

$DT_{sw}$ CCM

$D$

$S_{on}$ $I_p$ $\Delta I_L$

$DT_{sw}$ DCM

Same static waveforms in VM and CM

Indirect duty ratio control

$V_{on}$ $D$ $I_p = \frac{V}{R_i}$ $V_{off}$

Direct duty ratio control

$V_{err}$
A VM-PWM Switch-Based Small-Signal Model

- Small-signal model built after sampled-data analysis

- Model accurately predicts subharmonic oscillations but is ac only

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A Current-Mode PWM Switch Model

- The large-signal model associates three simple current sources

![Diagram of a current-mode PWM switch model]

- It can compute a bias point and accepts transient simulations
- The resonant tank reproduces subharmonic oscillations

\[ H(s) \approx H_0 \frac{1 + \frac{s}{\omega_p}}{1 + \frac{s}{\omega_n} + \left( \frac{s}{\omega_n} \right)^2} \]

\[ V_{in} \quad \frac{V_{cp} I_c}{V_{ap}} \quad \frac{V_c}{R_i} I_\mu \quad I_c \quad L \quad C \quad R \quad \text{resonant tank} \]

Same result with that obtained with sampled-data analysis.

V. Vorpérian, Analysis of Current-Controlled PWM Converters using the Model of the PWM Switch, PCIM Conference, 1990
Implementing the CM-PWM switch Model

- You cannot beat the CM-PWM switch model in terms of simplicity

Only these parameters are needed

- It is a large-signal model which also predicts transient response
The pulse width modulator is a highly nonlinear structure.

- Harmonics are fed back through the loop.
- Additional perturbations go through PWM.
- Aliasing effects are observed.

Predicting Phase Lag Approaching \( F\text{sw}/2 \)

- Modulation and sidebands go through the low-pass filter

\[
\begin{align*}
V_{in} & \\ \text{modulation} & \\ t_{on} & \\ L_1 & \\ C_2 & \\ R_L & \\ v_{out} & \\ \hat{v}_d & \\
\end{align*}
\]

- Low-pass filter

\[
\begin{align*}
- F_{sw} & \\ - F_{sw} + f_{mod} - f_{mod} & \\ f_{mod} & \\ F_{sw} - f_{mod} + f_{mod} & \\
\end{align*}
\]

- Linear LP response

\[
\begin{align*}
\hat{v}_{out} & \\
\end{align*}
\]

- Sideband is no longer negligible

F. Lee, Review of Current-Mode-Control Modeling, APEC 2017 Professional Education Seminars, Tampa, FL
Accounting for Sidebands Effects

- Harmonics are responsible for the phase deviation as \( f_{mod} \) approaches \( F_{sw}/2 \)

\[
\frac{V_{out}(s)}{V_c(s)} = \frac{F_{sw}}{(S_n + S_e) + (S_f - S_e)e^{-sT_{sw}}} \frac{R_{load}V_{in}}{sL} \left(1 + \frac{s}{\omega_z} \left(1 - e^{-sT_{sw}}\right)\right)
\]

Yingyi Yan’s model is a universal subcircuit covering various operating schemes. Need to feed the model with static parameters.

Same performance as CM-PWM switch but with a more complex implementation.

Five Different Control Schemes can be Analyzed

Replace some parameters with new expressions and simulate

**Constant On-Time:**

\[ C_e = \frac{t_{on}^2}{(L\pi^2)} \quad R_e = \frac{2L}{t_{on}} \]

\[ K_{ap} = \frac{t_{off}}{t_{on}} \]

\[ \hat{v}_{cp} G_{cp} \]

\[ \hat{v} L G_L \]

\[ \begin{array}{cccc}
R_{ap} & + & \hat{v}_{ap} K_{ap} & + \\
\hat{v}_{cp} & - & R_e & + \\
\hat{v} R_e & - & \hat{v} L & - \\
C & L & C' & L
\end{array} \]

**Constant Off-Time:**

\[ C_e = \frac{t_{off}^2}{(L\pi^2)} \quad R_e = \frac{2L}{t_{off}} \]

\[ K_{ap} = -1 \]

**Charge Control:**

\[ C_e = T_{sw}^2 \left( \frac{L\pi^2}{2} \right) \quad R_e = L \left[ T_{sw} \left( \frac{L_{L}}{V_{cp}T_{sw}} - \frac{D}{2} + \frac{S_e C_T}{S_f T_{sw}} \right) \right] \]

\[ K_{ap} = \left( \frac{t_{off}}{2L} \right) R_e \]

**Peak-Current Mode Control:**

\[ C_e = T_{sw}^2 \left( \frac{L\pi^2}{2} \right) \quad R_e = L \left[ T_{sw} \left( \frac{S_n + S_e}{S_n + S_f} - 0.5 \right) \right] \]

\[ K_{ap} = -\left( \frac{t_{off}}{2L} \right) R_e \]

**Valley-Current Mode Control:**

\[ C_e = T_{sw}^2 \left( \frac{L\pi^2}{2} \right) \quad R_e = L \left[ T_{sw} \left( \frac{S_n + S_e}{S_n + S_f} - 0.5 \right) \right] \]

\[ K_{ap} = \left( \frac{t_{on}}{2L} \right) R_e \]
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The PWM Switch in Current-Mode Control

- Determine a time-continuous equation linking variables

\[ \langle i_a(t) \rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}} \int_0^{dT_{sw}} i_a(t) dt = D \langle i_c(t) \rangle_{T_{sw}} = DI_c \]

\[ I_a = DI_c \]
Accounting for Compensation Ramp Effects

- The compensation ramp reduces the effective peak current

\[ I_{\text{peak}} \]

- Current at point b is that of a minus half the inductor ripple

\[ \langle i_c(t) \rangle_{T_{\text{sw}}} = \frac{V_c}{R_i} - \frac{S_a}{R_i} DT_{\text{sw}} \left( \frac{S_2 D'T_{\text{sw}}}{2} \right) \]
The inductor downslope \( S_2 \) in a buck is defined by \( S_2 = \frac{V_{out}}{L} \)

The downslope depends on the output voltage \( V_{out} \):
\[
S_2 = \frac{V_{out}}{L}
\]

The inductor average voltage is 0 V at steady-state: \( V_{cp} = V_{out} \)
Associating Current Sources

- Update the previous equation to obtain the final definition

\[
I_c = \frac{V_c}{R_i} - V_{cp} (1 - D) \frac{T_{sw}}{2L} - \frac{S_a}{R_i} DT_{sw}
\]

Peak current setpoint  Half inductor ripple  Compensation ramp

Group 2\textsuperscript{nd} and 3\textsuperscript{rd} terms

- Inductor ripple and compensation ramp alter peak value
Determining the Duty Ratio

- Voltage relationship between $v_{ap}$ and $v_{cp}$ holds in current mode

$$V_{cp} = DV_{ap}$$

$${\langle v_{ap} (t) \rangle}_{T_{sw}} = V_{ap} = \frac{1}{T_{sw}} \int_0^{D T_{sw}} v_{ap} (t) \, dt = D T_{sw} = D V_{ap}$$
Predicting Sub-Harmonic Oscillations

- The model, as it is, cannot predict instabilities
- Let's observe a small-signal perturbation in $v_c$

The off-slope does not change as $\hat{v}_{cp}$ keeps constant
- This "memory" effect is modeled with a capacitor $C_s$

\[ V_c(t) \]

\[ \hat{v}_{cp} = \frac{V_{cp}}{L} \]

\[ i_c(t) \]

\[ T_{sw} \]

\[ F_{sw} = \frac{1}{2\pi \sqrt{LC_s}} \]

\[ C_s = \frac{1}{L\left(F_{sw}\pi\right)^2} \]
Small-Signal Model of the CM-PWM Switch

- You can perturb...

\[ I_c + \dot{I}_c \quad V_c + \dot{V}_c \quad V_{cp} + \dot{V}_{cp} \quad V_{ap} + \dot{V}_{ap} \]

\[ I_c = \frac{V_c}{R_i} - V_{cp} \left(1 - D\right) \frac{T_{sw}}{2L} - \frac{S_a}{R_i} DT_{sw} \]

...or apply partial differentiation

\[ \dot{I}_c = \frac{\partial I_c}{\partial V_c} \dot{V}_c + \frac{\partial I_c}{\partial V_{ap}} \dot{V}_{ap} + \frac{\partial I_c}{\partial V_{cp}} \dot{V}_{cp} \]

\[ \dot{I}_a = \frac{\partial I_a}{\partial V_c} \dot{V}_c + \frac{\partial I_a}{\partial V_{ap}} \dot{V}_{ap} + \frac{\partial I_a}{\partial V_{cp}} \dot{V}_{cp} \]

- Laplace Transform now holds on this linear network
In a constant on-time circuit, the loop controls the valley current.

\[
\frac{\text{ms} L}{\text{wo} I} = D \quad \text{and} \quad \text{wo} I + \text{ms} l = \frac{\text{ms} L}{\text{wo} I - 1} \frac{\partial I}{\partial t} = l
\]

\[
\frac{\text{ms} L}{\text{wo} I} = 1 \quad \frac{\partial I}{\partial t} = l \quad \Rightarrow \quad \text{wo} I + \text{ms} l = \frac{\text{ms} L}{\text{wo} I - 1} \frac{\partial I}{\partial t} = l
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\[
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\]
We compare the cycle-by-cycle response with that of the averaged model.

Parameters

- $R_i = 100\, \text{m}$
- $i = 10\, \text{u}$
- $G = C/i$
- $L = 5\, \mu$
- $\text{ton} = 2.3\, \text{u}$
- $C = i/\text{ton}/1$

Transient responses are identical

Keeps the switch closed for a fixed $t_{on}$.

Sets the latch back high when valley current set by $V_c$ is reached.
SIMPLIS lets us obtain the dynamic response in a few seconds

Dynamic responses are identical
The PWM Switch Model in a FOT Converter

- In a fixed-off-time circuit, the loop controls the peak current

\[
I_c = \frac{V_c}{R_i} \frac{\Delta I_L}{2} = \frac{V_c}{R_i} \frac{V_{ac}}{2L} t_{on} \quad \text{and} \quad t_{on} = \frac{\left(\frac{V_c}{R_i} - I_c\right) 2L}{V_{ac}} \quad \text{is fixed} \\
I_\mu = \frac{V_{cp}}{2L} t_{off} \quad T_{sw} = t_{on} + t_{off} \quad I_c = \frac{V_c}{R_i} - I_\mu \quad D = \frac{t_{on}}{T_{sw}}
\]
Comparing the Cycle-by-Cycle Transient Waveforms

- We compare the cycle-by-cycle response with that of the averaged model.

Parameters:
- \( R_i = 50 \, \text{m}\Omega \)
- \( I_i = 10 \, \mu\text{A} \)
- \( G = C / i \)
- \( L = 5 \, \mu\text{H} \)
- \( t_{\text{off}} = 5 \, \mu\text{s} \)
- \( C = i \cdot t_{\text{off}} / 1 \)

Keeps the switch open for a fixed \( t_{\text{on}} \).
Resets the latch when the peak current set by \( V_c \) is reached.

Transient responses are identical.
SIMPLIS® easily simulates the fixed-off-time converter

Dynamic responses are very close
Quasi-Square-Wave Converters

The waveform average values do not include the deadtime

$\langle I_c(t) \rangle_{T_{sw}} = \frac{I_{peak}}{2}$

Derive operating points and dc transfer function

From Large- to Small-Signal Models

- Partial differentiation will give small-signal coefficients

\[ I_c = f(V_c) \rightarrow \hat{i}_c = \frac{\partial I_c(V_c)}{\partial V_c} \hat{v}_c \quad \hat{i}_c = \hat{v}_c \left( \frac{1}{2R_i} \right) = \hat{v}_c k_c \quad k_c = \frac{1}{2R_i} \]

\[ \hat{i}_a = \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial V_{cp}} \bigg|_{I_c, V_{ac}} \hat{v}_{cp} + \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial I_c} \bigg|_{V_{cp}, V_{ac}} \hat{i}_c + \frac{\partial I_a(V_{cp}, V_{ac}, I_c)}{\partial V_{ac}} \bigg|_{I_c, V_{cp}} \hat{v}_{ac} \]

\[ \hat{i}_a = \hat{v}_{cp} k_{cp} + k_{ic} \hat{i}_c - \hat{v}_{ac} k_{ac} \]

- Static coefficients

\[ k_{cp} = \frac{I_{c0} V_{ac0}}{(V_{ac0} + V_{cp0})^2} \]

\[ k_{ic} = \frac{V_{cp0}}{V_{cp0} + V_{ac0}} \]

\[ k_{ac} = \frac{V_{cp0} I_{c0}}{(V_{ac0} + V_{cp0})^2} \]
Cycle-by-Cycle and Averaged Responses

The cycle-by-cycle circuit requires an extra winding to detect core reset.
The switching frequency varies with line and load conditions.

Cycle-by-cycle results

1st-order dynamic response

public information
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The Fastest Way to Determine a Transfer Function

- We can use the Fast Analytical Circuits Techniques or FACTs
  \[
  \frac{V_{\text{out}}(s)}{D(s)} = H_0 \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2}
  \]
  \[
  a_1 \text{ and } b_1 \begin{bmatrix} s \end{bmatrix}
  \]
  \[
  a_2 \text{ and } b_2 \begin{bmatrix} s^2 \end{bmatrix}
  \]
  Nulled response: \[a_1 = \tau_{1N} + \tau_{2N}\]
  Zeroed excitation: \[b_1 = \tau_1 + \tau_2\]

- Energy-storing elements are combined with resistances
  \[+\]
  \[\tau = \frac{L}{R}\]
  \[\tau = RC\]
  Time constants

- Capacitors and inductors behave differently for \(s = 0\) and \(s \to \infty\)

\[s \to \infty \quad C \quad s = 0\]
\[L \quad s \to \infty \quad s = 0\]
Determine the Dc Gain

- Look at the circuit for \( s = 0 \)
  - Capacitor are open circuited
  - Inductors are short circuited

\[
\text{SPICE operating point calculation}
\]

\[
V_{\text{out}} = V_{\text{in}} \left( R_1 + r_C \right)
\]

Determine the gain in this condition:

\[
H_0 = \frac{R_{\text{load}}}{R_{\text{load}} + R_1}
\]
Determine the Time Constant

- Look at the resistance driving the energy-storing element
  1. When the excitation is turned off, \( V_{in} = 0 \text{ V} \)

- Remove the capacitor and “look” into its terminals
  - The first time constant is \( \tau_1 = (r_C + R_1 \parallel R_{load})C_1 \)
Null the Output to Unveil the Zero Location

- Bring the excitation back

2. Check the condition bringing $V_{out} = 0 \text{ V}$

$$Z_1(s) = r_c + \frac{1}{sC_1} = 0$$

You can also remove the capacitor and look into its terminals

- The second time constant in the null condition is $\tau_2 = r_c C_1$
Combine the Time Constants in a Low-Entropy Form

By combining time constants, we have

\[ H(s) = H_0 \frac{1 + s\tau_2}{1 + s\tau_1} = \frac{R_{load}}{R_{load} + R_1} \frac{1 + sr_c C_1}{1 + s(r_c + R_1 \parallel R_{load})C_1} \]

Rearrange the equation to unveil a pole and a zero

\[ H(s) = H_0 \frac{1 + \frac{s}{\omega_p}}{1 + \frac{s}{\omega_z}} \]

\[ \omega_z = \frac{1}{r_c C_1} \quad H_0 = \frac{R_{load}}{R_{load} + R_1} \]

\[ \omega_p = \frac{1}{(r_c + R_1 \parallel R_{load})C_1} \]

This is a low-entropy expression
Second-Order Circuits: New Notation

- Set one reactance into its high-frequency state
  - Reactance 1 is in its high-frequency state
  - What resistance drives reactance 2?
  - Reactance 2 is in its high-frequency state
  - What resistance drives reactance 1?

- There is redundancy: pick the simplest result
  \[ b_2 = \tau_1 \tau_2 \quad \text{or} \quad b_2 = \tau_2 \tau_1 \]
A Simple Low-Pass $LC$ Filter

- There are two energy-storing elements: 2\textsuperscript{nd}-order filter

- What transfer function links $V_{\text{out}}$ to $V_{\text{in}}$?

Set $s$ to zero: short inductors, open caps.

- Determine $H_0$

Reduce excitation to 0 V: short $V_{\text{in}}$

- Find 1\textsuperscript{st}-order time constants with $L_1$ and $C_2$

- Find 2\textsuperscript{nd}-order time constants with $L_1$ and $C_2$

Null the output and find zero(es)

Assemble results in a low-entropy form

$$H(s) = H_0 \frac{1 + \frac{s}{\omega_0}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$
Start with the Dc Gain

- Short the inductor and open the capacitor

\[ V_{\text{out}} = V_{\text{load}} \]

\[ H_0 = \frac{R_{\text{load}}}{R_{\text{load}} + r_L} \]

- This expression is obtained by inspecting the circuit: no algebra
Reduce the Excitation to 0 V: Short $V_{in}$

- Determine the 1$^{st}$-order time constants

\[ \tau_1 = \frac{L_1}{R_{load} + r_L} \]

\[ \tau_2 = \left( r_C + r_L \parallel R_{load} \right) C_2 \]

Don’t expand!
Pick the Simples Time Constant Combination

- Set one of the energy-storing elements in its high-frequency state

\[ \tau_1^2 = \frac{L_1}{r_L + r_C \parallel R_{load}} \]

\[ b_2 = \tau_2 \tau_1^2 \leftarrow \rightarrow b_2 = \tau_1 \tau_2^1 \]

\[ \tau_2^1 = C_2 \left( r_C + R_{load} \right) \]

- You have to select the simplest combination between the two options
What would prevent the stimulus \( V_{in} \) from forming a response \( V_{out} \)?

There is one single zero contributed by \( C_2 \)

\[
Z_1(s) = sL_1 + r_L \to \infty
\]

Only for \( s \) approaching infinity

\[
Z_2(s) = \frac{1}{sC_2} + r_C = 0
\]

\[
s_z = -\frac{1}{r_C C_1}
\]

\[
\omega_z = \frac{1}{r_C C_1}
\]
Assemble all Time Constants to form $H(s)$

- The denominator is obtained by combining the time constants

$$D(s) = 1 + b_1 s + b_2 s^2 = 1 + s \left[ \frac{L_1}{R_{load} + r_L} + (r_C + r_L \ || R_{load}) C_2 \right] + L_1 C_2 \frac{r_C + R_{load}}{R_{load} + r_L} \cdot s^2$$

- The numerator features one zero only

$$N(s) = 1 + s r_C C_2$$

- Read gains, poles and zero with the low-entropy format

$$H(s) = H_0 \left( 1 + \frac{s}{\omega_z} \right)^2 \frac{1 + \frac{s}{\omega_0 Q} + \left( \frac{s}{\omega_0} \right)}{1 + \frac{s}{\omega_0}} \quad \omega_z = \frac{1}{r_C C_2} \quad \omega_0 = \frac{1}{\sqrt{L_1 C_2}} \quad H_0 = \frac{R_{load}}{r_L + R_{load}}$$

$$Q = \frac{r_L + R_{load}}{L_1 + C_2 \left[ r_C r_L + R_{load} (r_C + r_L) \right]} \frac{1}{\omega_0}$$

R. D. Middlebrook, Methods of Design-Oriented Analysis: Low Entropy Expressions, New Approaches to Undergraduate Education, July 1992
Check Results and Easily Run Corrections if needed

- Verify the expression in a Mathcad® sheet and compare responses

\[ r_L := 0.1 \Omega \quad r_C := 1 \Omega \quad C_2 := 1 \mu F \quad L_1 := 20 \mu H \quad R_{\text{load}} := 5 \Omega \quad \|(x, y) := \frac{xy}{x+y} \]

- If a deviation exists, fix the guilty sketch and don’t restart from scratch
Third-Order Transfer Function: more Choice!

- Combine the three low-frequency times constants together

\[ \begin{align*}
\tau_{13} & \quad \rightarrow \quad \text{Reactance 1 and 3 are set in their high-frequency state} \\
\tau_{2} & \quad \rightarrow \quad \text{What resistance drives reactance 2?} \\
\tau_{23} & \quad \rightarrow \quad \text{Reactance 2 and 3 are set in their high-frequency state} \\
\tau_{1} & \quad \rightarrow \quad \text{What resistance drives reactance 1?}
\end{align*} \]

\[ D(s) = 1 + s(\tau_1 + \tau_2 + \tau_3) + s^2(\tau_1\tau_2^1 + \tau_1\tau_3^1 + \tau_2\tau_3^2) + s^3\tau_1\tau_2\tau_3^{12} \]

C. Basso, Linear Circuits Transfer Functions: An Introduction to Fast Analytical Techniques, Wiley 2016
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There are four open-loop transfer functions of interest:

\[ \frac{V_{out}(s)}{V_c(s)} \bigg|_{\hat{v}_{\text{in}}=0} \]

Control-to-output with ac-silent input source

“How does a voltage perturbation on \(v_c\) propagate to \(v_{out}\)?”

\[ \frac{V_{out}(s)}{V_{in}(s)} \bigg|_{\hat{v}_c=0} \]

Input-to-output with static control \(V_c\)

“How does a voltage perturbation on \(v_{in}\) propagate to \(v_{out}\)?”

\[ \frac{V_{out}(s)}{I_{out}(s)} \bigg|_{\hat{v}_{\text{in}}=\hat{v}_c=0} \]

Output impedance with static control \(V_c\)

“How does a current perturbation on \(i_{out}\) propagate to \(v_{out}\)?”

\[ \frac{V_{in}(s)}{I_{in}(s)} \bigg|_{\hat{v}_c=0} \]

Input impedance with static control \(V_c\)

“How does a current perturbation on \(i_{in}\) propagate to \(v_{in}\)?”
Control-to-Output Transfer Function

- Install the small-signal CM-PWM switch in the buck converter

- We want the control-to-output function, the input source is ac-silent:

\[ \hat{v}_{in} = 0 \quad \Rightarrow \quad \hat{v}_{ap} g_f = 0 \]

- The input contribution can also disappear, no interest in \( Z_{in} \) for now
Simplify and Rearrange the Circuit

- The circuit now looks simpler to study

Assign labels to identify time constants: $\tau_1$, $\tau_2$, $\tau_3$

- There are 3 energy-storing elements: 3$^{rd}$-order system
Look at each Time Constant when Excitation is Off

- The excitation is zero, elements are in their dc states
- 3 storage elements, 3 time constants, 3 drawings

\[ \tau_1 = f(C_1) \quad \rightarrow \quad \tau_1 = C_1 \left( \frac{1}{g_o} \right) \]

\[ \tau_2 = f(L_2) \quad \rightarrow \quad \tau_2 = \frac{L_2}{1 + \frac{1}{g_o} + R} \]

\[ \tau_3 = f(C_3) \quad \rightarrow \quad \tau_3 = C_3 \left( r_C + \left( \frac{1}{g_o} \right) \right) \]

Sum time constants:
Dimension is time [s]

\[ b_1 = \tau_1 + \tau_2 + \tau_3 \]

\[ b_1 = C_1 \left( \frac{1}{g_o} \right) + \frac{L_2}{1 + \frac{1}{g_o} + R} + C_3 \left( r_C + \left( \frac{1}{g_o} \right) \right) \]

Control to output
Determine Second-Order Coefficients

- One element is now set in its high-frequency state

$$\tau_2^1 = \frac{L_2}{R}$$

$$\tau_3^1 = r_C C_3$$

$$b_2 = \tau_1 \tau_2^1 + \tau_1 \tau_3^1 + \tau_2 \tau_3^2 \left[ s^2 \right]$$

$$b_2 = C_1 \left( \frac{1}{g_o} \right) \frac{L_2}{R} + C_1 \left( \frac{1}{g_o} \right) r_C C_3 + \frac{L_2}{1 + R} (r_C + R) C_3$$

Control to output
Two Elements are now set in High-Frequency State

- For $b_3$, we multiply by a third time-constant:
  \[ b_3 = \tau_1 \tau_2 \tau_3 \quad \text{Dimension is time}^3 \]

- What is this new time constant definition?

- The final coefficient has been identified:
  \[ b_3 = C_1 \left( \frac{1}{g_o} \right) \left( \frac{L_2}{R} \right) (r_C + R) C_3 \]
Assemble Time Constants to Build the Transfer Function

- A Mathcad® sheet can be built to verify these calculations

\[
H(s) = G_0 \left( \frac{1 + \frac{s}{S_{z_1}}}{1 + b_1 s + b_2 s^2 + b_3 s^3} \right)
\]

\[
G_0 = k_0 \left( R \left\| \frac{1}{g_0} \right\| \right)
\]

- \( \omega_{z_1} = \frac{1}{r_C C} \)

- \( b_1 = C_1 \left( \frac{1}{g_o} \right) \frac{L_2}{1 + g_0 R} + C_3 \left( r_C + \left\| \frac{1}{g_0} \right\| R \right) \)

- \( b_2 = C_1 \left( \frac{1}{g_o} \right) \frac{L_2}{R} + C_1 \left( \frac{1}{g_o} \right) r_C C_3 + \frac{L_2}{1 + g_0 R} \left( r_C + R \right) C_3 \)

- \( b_3 = C_1 \left( \frac{1}{g_o} \right) \frac{L_2}{R} (r_C + R) C_3 \)

5 V/1 A buck

- \( V_{in} = 10 \text{ V}, F_{sw} = 100 \text{ kHz}, R_i = 0.25 \Omega, S_e = 2.5 \text{ kV/s} \)
- \( C = 100 \mu \text{F}, r_C = 0.1 \Omega, L = 100 \mu \text{H}, C_s = 101 \text{nF}, V_c = 1.28 \text{ V} \)
- \( I_c = 4.94 \text{ A} \)
- \( k_i = 2 \Omega^{-1}, k_0 = 4 \Omega^{-1} \)
- \( g_0 = 0.01 \Omega^{-1} \)
- \( g_f = -7.5 \text{ m\Omega}^{-1}, g_r = 0.49 \Omega^{-1} \)
- \( g_i = -250 \text{ m\Omega}^{-1} \)

- \( G_0 = 12 \text{ dB} \)
- \( f_{z_1} = 15.9 \text{ kHz} \)
Verify the Analysis with a SPICE Simulation

- We can use the large-signal model to confront our mathematical results

\[ \text{parameters} \]
- Fsw = 100kHz
- L = 100u
- Cs = 1/(L*(Fsw*3.14)^2)
- Ri = 250m
- Se = 2.5k

\[ \text{Bias points} \]
- + V1 = 10V
- R3 = 1m
- p
- p
- 1.28V
- + Vstim 1.28
- AC = 1
- B1 Voltage v(c,p)/v(a,p)

\[ \text{Check operating points are correct: } V_{out} = 4.95 \text{ V} - \text{ ok} \]
Excellent Agreement between SPICE and Equations

- Superimposed curves means transfer functions are identical.
Rearrange the Expression in a *Low-Entropy* Form

- Proper arrangement is necessary to gain insight in the expression

\[ D(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3 \]

- This is a third-order polynomial form that can be factored

\[ H(s) \approx H_0 \frac{1}{1 + \frac{s}{\omega_{z_1}}} \left[ 1 + \frac{s}{\omega_p} \right] \left[ 1 + \frac{s}{\omega_Q} \right] + \left( \frac{s}{\omega_n} \right)^2 \]
Final Expression shows Poles and Zeroes

- The transfer function can now unveil peaking and damping

\[
H_0 = \frac{R}{R_i} \frac{1}{1 + \frac{RT_{sw}}{L_2} \left[ m_c (1 - D) - 0.5 \right]}
\]

\[
\omega_{p_1} = \frac{1}{RC_3} + \frac{T_{sw}}{L_2 C_3} \left[ m_c (1 - D) - 0.5 \right] \quad \omega_{z_1} = \frac{1}{r_c C_3}
\]

\[
\omega_n = \frac{\pi}{T_{sw}} \quad Q = \frac{1}{\pi \left[ m_c (1 - D) - 0.5 \right]}
\]

\[
m_c = 1 + \frac{S_e}{S_n} \quad \text{Artificial ramp}
\]

\[
m_c = 1 + \frac{S_e}{S_n} \quad \text{Inductor on-slope}
\]

The control voltage is reduced to 0 V and $V_{in}$ is now ac-modulated.

We have no interest in the left-side part now as it affects $Z_{in}$ only.

In open-loop conditions, $\hat{v}_c(s)$ is 0 V and it simplifies the circuit.
The Dc Gain is Immediate

- When the excitation is zeroed, the circuit returns to its natural state
- Reuse the denominator you have already determined

\[ H(s) \approx H_0 \left( \frac{1 + \frac{s}{\omega z_1}}{1 + \frac{s}{\omega p_1}} \right) \frac{1}{1 + \frac{s}{\omega_n Q} + \left(\frac{s}{\omega_n}\right)^2} \]

\[ H_0 = g_f \left( \frac{1}{g_o} \parallel R_{load} \right) \]

\[ \omega z_1 = \frac{1}{r_c C_3} \]

- Current-mode control naturally rejects the input contribution as \( H_0 \) is low
Rejecting the Input Voltage Contribution

Calculate the amount of external ramp to reject $V_{in}$ perturbations

$$H_0 \approx \frac{D \left[ m_c (1 - D) - \left( 1 - \frac{D}{2} \right) \right]}{L_2 R_{\text{load}} T_{\text{sw}}} + \left[ m_c (1 - D) - 0.5 \right]$$

$$D \left[ m_c (1 - D) - \left( 1 - \frac{D}{2} \right) \right] = 0$$

$$m_c = \frac{D - 2}{2D - 2}, \quad S_e = 50\% \cdot S_n$$

$D = 50\%$

✓ Theoretical infinite input rejection!
Output Impedance Determination

- Install a current source across the load resistance

\[ V_{in}(s) \uparrow \frac{1}{g_i} \downarrow \hat{v}_c k_i \quad \hat{v}_{cp} g_r \quad \hat{v}_{in} \quad \hat{v}_{ap} g_f \quad \hat{v}_c k_o \quad \frac{1}{g_o} \downarrow C_1 \quad C_3 \quad I_c(s) L_2 \quad I_T \quad V_T \]

- The two controlled sources are turned off as \( V_{in} \) and \( V_c \) are 0 V

\[ V_{out}(s) = \frac{V_T(s)}{I_T(s)} \]

Draw a simpler sketch

Adjust the current source to determine \( I_T \) and \( V_T \)
Determine Time Constants by Inspection

- Short the inductor and open the capacitor for $R_0$

$$s = 0$$

\[ \frac{1}{g_0} \]

$$L_2$$

$$C_3$$

$$R_{load}$$

$$R?$$

Turn excitation off

$$L_2$$

$$C_3$$

$$R_{load}$$

$$I_T = 0$$

Back to the natural structure

Reuse previous denominator

$$D(s) \approx \left[1 + \frac{s}{\omega_p^1}\right] \left[1 + \frac{s}{\omega_n Q} + \left(\frac{s}{\omega_n}\right)^2\right]$$

Output impedance
Null the Response for the Zeroes

- The response is nulled when no current circulates in $R_{load}$.

\[
Z_1(s) = r_c + \frac{1}{sC_3} = 0 \quad \omega_z = \frac{1}{r_cC_3}
\]

1st zero

No current flows in $R_{load}$

All the $I_T$ current circulates in $L_2$

The impedance involving $L_2$, $C_1$ and the conductance is a transformed short circuit.

\[
Z_2(s) = sL_2 + \frac{1}{g_o} \parallel \frac{1}{sC_1} = 0
\]

\[
Z_2(s) = \frac{1 + s g_o L_2 + s^2 L_2 C_1}{D(s)} = 0
\]

\[
N(s) = 1 + \frac{s}{\omega_N Q_N} + \left(\frac{s}{\omega_N}\right)^2
\]
Simplify and test the Final Expression

The second-order polynomial form is affected by \( Q \) and \( \omega_0 \)

\[
\omega_N = \frac{1}{\sqrt{L_2C_1}} \quad Q_N = \frac{1}{g_o \sqrt{L_2}}
\]

High-frequency contribution

\[
Z_{out}(s) = R_0 \frac{1 + s/\omega_z}{1 + \frac{s}{\omega_p} + \left( \frac{s}{\omega_N} \right)^2}
\]

\[
Z_{out}(s) \approx \frac{R_{load}}{1 + \frac{R_{load}}{L_2 T_{sw} \left[ m_c (1 - D) - 0.5 \right]}} \frac{1 + s/\omega_z}{1 + \frac{s}{\omega_p}}
\]

The impedance is dominated by \( R_{load} \) in low frequency

Need high open-loop gain to lower \( Z_{out} \)

Output impedance
Input Impedance Determination

- The excitation source now biases the input port

- Determine the incremental resistance $R_0$ for $s = 0$
A Negative Open-Loop Incremental Resistance

The input current is the test current $I_T$

Replace coefficients by their definition and rearrange

The incremental resistance of the CCM open-loop buck converter is negative
Turn the Excitation off and Determine Time Constants

- Get rid of the controlled sources for an easy inspection

\[
\begin{align*}
V_{(a)} &= -\frac{1}{g_i} V(c, p) g_r \\
\hat{v}_{cp} &= \frac{1}{R} \\
\hat{i} &= \hat{v}_{cp}
\end{align*}
\]

- Inspection works to determine the time constants

\[
\frac{g_i}{g_r g_f} \quad \frac{1}{g_o} \quad r_c
\]

Determine resistances

3rd-order circuit
Break the Circuit into Small Individual Sketches

$$\tau_1 = C_1 \left( \frac{1}{g_o} \ || \ R_{load} \ || \ \frac{g_i}{g_r g_f} \right)$$

$$\tau_2 = \frac{L_2}{\frac{1}{g_o} \ || \ \frac{g_i}{g_r g_f} + R_{load}}$$

$$\tau_3 = C_3 \left( r_C + \frac{1}{g_o} \ || \ R_{load} \ || \ \frac{g_i}{g_r g_f} \right)$$

Input impedance

\[ \tau_2^1 = \frac{L_2}{R_{load}} \]

\[ \tau_3^1 = C_3 r_C \]

\[ \tau_3^2 = C_3 \left( r_C + R_{load} \right) \]
Combine the Time Constants

- Assemble the time constants to form the denominator:

\[
D(s) = 1 + s^2b_1 + s^2b_2 + s^3b_3
\]

\[
b_1 = \tau_1 + \tau_2 + \tau_3
\]

\[
b_2 = \tau_1^1.r_2 + \tau_1^1.r_3 + \tau_2^2
\]

\[
b_3 = \tau_1^1.r_2^1 + \tau_3
\]

\[
\tau_3^2 = C_3(r_C + R_{\text{load}})
\]

- 0-V \( V_T \) brings the circuit back to its structure: reuse former \( D(s) \) of slide 87

\[
V_T = 0 \text{ V}
\]

Degenerate case

Same \( \tau \)

See slide 87
Plot the Open-Loop Input Impedance

The input impedance is negative up to high frequencies

\[ Z_{in}(s) = R_0 \frac{N(s)}{D(s)} \]

\[ N(s) = 1 + a_1 s + a_2 s^2 + a_3 s^3 \]

\[ D(s) = 1 + b_1 s + b_2 s^2 + b_3 s^3 \]

See slide 87

\[ |Z_{in}(f)| \]

\[ \angle Z_{in}(f) \]
Why is the Incremental Resistance Negative?

- The smaller on-time at high line offers more demagnetization time
- The valley current is lower: $I_{out}$ decreases when $V_{in}$ increases

A voltage increase brings an input current decrease:

$$R_0 := \frac{V_1 + V_{step} - V_1}{I_{in}(V_1 + V_{step}) - I_{in}(V_1)} = -12.61 \Omega$$

$$V_1 = 9 \text{ V} \quad V_{step} = 1 \text{ mV}$$
Unstable in Open-Loop Conditions with an EMI Filter

- For many converters, filter-related instabilities occur in closed loop.
- The CCM CM buck can be unstable in open-loop conditions.

- It is a simple open-loop configuration, 5 V/5 A.
The Filter Output is Affected by a High Ripple

The converter can barely deliver the voltage but its input is highly unstable.

Filter damping is an absolute necessity!
The Filter Output Impedance brings the Problem

- Stability can be at stake when inserting the filter

\[ V_{th}(s) \]
\[ Z_{th}(s) \]
\[ V_{in}(s) \]
\[ Z_{in}(s) \]

- The circuit can be modeled to reveal a minor loop

\[ V_{th}(s) \]
\[ Z_{th}(s) \]
\[ Z_{in}(s) \]
\[ V_{in}(s) = V_{th}(s) \frac{1}{1 + \frac{Z_{th}(s)}{Z_{in}(s)}} \]

The Nyquist criterion applies

\[ \frac{Z_{th}(s)}{Z_{in}(s)} = -1 \]

\[ \left| \frac{Z_{th}(s)}{Z_{in}(s)} \right| = 1 \text{ and } \angle \frac{Z_{th}(s)}{Z_{in}(s)} = -180^\circ \]

Stay away from overlaps

Check EMI Filter Output Impedance Peaking

- Does overlap exist with the negative incremental resistance?
  - Yes – damp the filter with external components

![Electrical schematic diagram of an EMI filter]

- \( R_2 = 50\,\text{m} \)
- \( R_7 = 22\,\text{m} \)
- \( V_{Z\text{OUT}} \)
- \( L_2 = 10\,\mu\)H
- \( C_3 = 4.7\,\mu\)F

Graph showing:
- Input impedance
- Conditions for oscillations
- Design target
- \( R_0 = 10\,\text{dBΩ} \)
Optimally Damp the Filter Output Impedance

- The target is to reduce the filter impedance peak to 0 dBΩ or 1 Ω

\[ R_0 = \sqrt{\frac{L_f}{C_f}} \]
\[ \left| \frac{Z_{out}}{R_0} \right|_{mm} = \sqrt{\frac{2(2+n)}{n^2}} \]
\[ Q_{opt} = \sqrt{\frac{(4+3n)(2+n)}{2n^2(4+n)}} = 0.506 \]
\[ n = \frac{R_0 \left( R_0 + \sqrt{R_0^2 + 4\left| Z_{out} \right|_{mm}^2} \right)}{\left| Z_{out} \right|_{mm} \left( \left| Z_{out} \right|_{mm} \right)^2} = 5.738 \]
\[ R_{damp} = R_0 Q_{opt} = 0.74 \Omega \]
\[ C_{damp} = nC_f = 27 \mu F \]

- This is a rather large capacitance value for a ceramic device
- A 47-µF electrolytic capacitor and its ESR can do the job
- Watch for temperature effects as ESR increases at low temp!

C. Basso, EMI Input Filter Interaction with Switching Converters, APEC Professional Seminars, 2017
The Peak is Lowered Owing to Extra Losses

- The added $RC$ network exactly meets the design target.
Filter Damping has calmed down Oscillations

- Once the damper is added, high-frequency ripple remains.

- Check margins again once the converter is compensated.

Input impedance
Reduce the Load Current to enter DCM

The control-to-output transfer function is still of second order!

\[ H(s) \approx H_0 \left(1 + \frac{s}{\omega_z}\right) \left(1 + \frac{s}{\omega_{p_1}}\right) \left(1 + \frac{s}{\omega_{p_2}}\right) \]

\[ \omega_{p_1} = \frac{1}{R_{load} C_1} \frac{2m_c - (2 + m_c)M}{m_c (1-M)} \]

\[ \omega_{p_2} = 2F_{sw} \left(\frac{M}{D}\right)^2 \]

Instability in Absence of Compensation Ramp

- When the ramp reduces to zero, the DCM CM buck can be unstable

\[ \omega_{p_i} = \frac{1}{R_{load}C_1} \frac{2m_c - (2 + m_c)M}{m_c(1 - M)} \]

Reduce \( m_c \) to 1

\[ \omega_{p_i} = \frac{1}{R_{load}C_1} \frac{2 - 3M}{1 - M} \]

- Plot the pole position versus \( M \)

- As \( M \) increases (\( V_{in} \) is lowered), the pole approaches the origin

- For \( M = 0.666 \), the pole is at the origin

- For \( M \) greater than 0.666, the pole jumps in the right half-plane

- Despite DCM, a compensation ramp is needed

\( f > 0 \)

\( f < 0 \)

\[ f(M_1) = \frac{3M_1 - 2}{M_1 - 1} \]
Course Agenda

- The Buck Converter
- Control Schemes
- Introduction to Modeling
- The PWM Switch in Current-Mode Control
- Introduction to the Fast Analytical Techniques
- The Four Transfer Functions in CCM Current-Mode Control
- Compensation Strategy
- Prototype Measurements
A nonlinear average model is the ideal tool

- Auto-bias the output to its operating point, $I_{out} = 10$ A

- Check all operating points are correct ($V_{out} = 5$ V)

- Extract magnitude and phase information from the Bode plot
Stabilizing the Current-Mode Buck Converter

What you need is the control-to-output transfer function of the power stage

\[ \frac{V_{\text{out}}(f)}{V_{\text{FB}}(f)} \]

- \( f_c = 5 \text{ kHz}, G = -12 \text{ dB}, \varphi = -70^\circ \)
- \( f_c = 10 \text{ kHz}, G = -17 \text{ dB}, \varphi = -62^\circ \)
- \( f_c = 20 \text{ kHz}, G = -20.7 \text{ dB}, \varphi = -49^\circ \)
Automate the Compensation Process with k-Factor

- Use a SPICE simulation to try different compensation scenarios

- Keep phase margin constant and adjust crossover frequency

---

Public Information
Step-Load the Output of the Converter in Closed Loop

- With a constant 60° phase margin, recovery slope is constant

![Diagram showing the output voltage response to a load step](image)

1. **Load the Output of the Converter**
2. **Closed Loop**

---

**Parameters**

- $f_c = 10$ kHz
- $f_c = 5$ kHz
- $f_c = 20$ kHz
- $I_{out} = 3$ A in 3 µs
- $i_{out}(t)$
- $v_{out}(t)$

---

**Dependence**

- Depends on $C_{out}$ and $f_c$
- $\Delta I_{out} = 3$ A in 3 µs

---

**Public Information**

2/2/2018

[ON Semiconductor](https://www.onsemi.com)
The phase margin affects the overshoot and the recovery time. Crossover is constant to 5 kHz.

Marginally depends on PM

\[ \Delta I_{out} = 3 \text{ A in 3 } \mu\text{s} \]
The phase margin selection depends on the transient response you want.

The open-loop phase margin affects the closed-loop quality factor.

C. Basso, *The Dark Side of Loop Control Theory*, APEC 2012 Professional Seminar
What Transient Response is Needed?

Choose phase margin based on the transient response you want

- $\varphi_m = 45^\circ$: Fast recovery, little overshoot
- $\varphi_m = 80^\circ$: Slow recovery, 0 overshoot

$\Delta I_{out} = 3$ A in $3 \mu$s
Crossover Selection for the Buck VM Converter

- Before selecting a value, there are limits to respect.

- You need loop gain at peaking!

- Crossover for a CCM VM buck must be: \(3 \cdot f_0 < f_c < \frac{F_{sw}}{2}\)

- In this example: \(f_c > 6 \text{ kHz}\)
Compensation in Current Mode is Easier

- Once the poles are damped, the *theoretical* crossover limit is $F_{sw}/2$

\[ H(f) \]
\[ \angle H(f) \]

\[ F_{sw}/2 = 125 \text{ kHz} \]

CCM CM buck, $F_{sw} = 250$ kHz

- Don’t push $f_c$ too far as it increases susceptibility to noise 🚫
- As $f_c$ goes up, beware of various delays (conversion time, prop. del.)

Choose crossover to meet transient response, not more!
Approximating the Transient Response

- The capacitor impedance at crossover dominates the output impedance.

\[ Z_{out,CL}(s) = \frac{Z_{out,OL}(s)}{1 + T(s)} \]

- As crossover increases, \( Z_{out} \) approaches the minimum set by \( r'_{C} \).

If \( C_{out} \)'s impedance dominates at \( f_{c} \) then

\[ \Delta V_{out} \approx \frac{\Delta I_{out}}{2\pi f_{c} C_{out}} \]

\[ r_{c} \ll \frac{1}{2\pi f_{c} C_{out}} \]
A Simple Guide to Crossover Selection

- Select the output capacitor based on ripple current, ESR etc.

- Choose the crossover frequency to meet undershoot specs

\[
\Delta V_{out} < 100 \text{ mV}
\]

\[
C_{out} = 560 \ \mu\text{F}
\]

\[
r_C = 5 \ \text{m}\Omega
\]

\[
f_c \approx \frac{\Delta I_{out}}{2\pi C_{out} \Delta V_{out}} = 15 \ \text{kHz}
\]

- It surely is an approximation as the system is nonlinear but it's a guide

CCM CM buck \( F_{sw} = 250 \ \text{kHz} \)

\( f_c = 15 \ \text{kHz} \)

\( \varphi_m = 70^\circ \)
Stabilizing the Buck Converter Operated in Voltage Mode

Display the control-to-output transfer function with SPICE, SIMPLIS® etc...

- Extract magnitude and phase data at 10 kHz: a type 3 is required
Place Poles and Zeros

- Place two coincident zeroes at the resonant frequency (1 kHz)
- Place the first pole to build phase margin
- Place the second pole at $F_{SW}/2$ to roll-off the gain at high frequency

parameters

- $R_{upper}=10k$
- $f_c=10k$
- $G_{fc}=-19.6$
- $p_{fc}=-143$
- $\pi=3.14159$
- $f_{z1}=1k$
- $f_{z2}=1k$
- $f_{p1}=fc/\tan((2*\tan(fc/fz1)-\tan(fc/fp2))-\text{boost}^\pi/180)$
- $f_{p2}=50k$
- $C_1=1/(2*p_{i}^*fz1*R2)$
- $C_2=C1/(C1*R2+2*p_{i}^*f_{p1}-1)$
- $C_3=(f_{p2}-f_{z2})/(2*p_{i}^*R_{upper}*f_{p2}*f_{z2})$
- $R_3=R_{upper}^*f_{z2}/(f_{p2}-f_{z2})$
Check the Loop Gain in CCM and DCM

- Reducing the load to 50 Ω forces discontinuous operation

- Loop gain changes in DCM: watch weak phase margin zone
Split the Zeroes to Improve the Phase Response

- Rather than two coincident zeroes, place one at $f_0$ and another at 200 Hz.

- Lowering the zero with a constant $f_c$ reduces the gain at low frequency.

---

**Diagram:**

1. **CCM**
   - $|T(f)|$
   - $\angle T(f)$
   - $\varphi_m = 70^\circ$
   - 23 dB at 10 kHz

2. **DCM**
   - $|T(f)|$
   - $\angle T(f)$
   - $\varphi_m = 86^\circ$
   - $\varphi = 54^\circ$ at 300 Hz

---
A Better Phase Margin in DCM Slows the Response

- More pronounced undershoot with split zeroes and longer recovery

![Graph showing transient response and phase margin changes](graph.png)

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Overshoot</th>
<th>Settling time</th>
<th>Phase margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{z_1}$</td>
<td></td>
<td></td>
<td>faster</td>
<td></td>
</tr>
<tr>
<td>$f_{z_2}$</td>
<td></td>
<td></td>
<td>slower</td>
<td>faster</td>
</tr>
</tbody>
</table>

- Adjusting the second zero position changes the transient response
Stabilizing the Buck Converter Operated in Current Mode

- Use the PWM switch to plot the control-to-output transfer function

![Diagram of the Buck Converter](image)

- Extract magnitude and phase data at 10 kHz: a type 2 is required.
You can use the k-factor method in CM designs

- Zero and pole are spread to boost the phase at crossover
- The pole at the origin provides dc gain to minimize the static error

Check the Loop Gain in CCM and DCM

- Crossover frequency does not significantly change in DCM

- Less loop gain impact in DCM: 20-dB margin in magnitude curve
Course Agenda

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- Compensation Strategy
- Prototype Measurements
Characterize your Components on the Bench

- Bench experiments are a mandatory step
  - Are the assumptions adopted for the analytical model confirmed?
  - Feed the model back with real measurements, refine simulations
  - Check the behavior in temperature, particularly stability

- Capacitance change with bias voltage?

- Margin before inductor saturation?
A Buck Converter in Voltage-Mode Control

- Turn the UC3843 into a voltage-mode controller

- The soft-start provides a smooth start-up sequence
- The OVP circuit protects the converter in open-loop operations
The Need for a Bootstrapped Driver

- The UC384x delivers a ground-referenced drive signal

The circuit brings the gate 8 V above $V_{cc}$ and enhances the MOSFET

L. Balogh, Design and Application Guide for High-Speed MOSFET Gate Drive Circuit, TI SLUA 618
The Prototype in Voltage-Mode Control

- The board allows the easy testing of various compensation strategies

- You can toggle between open- and closed-loop configurations
Simulation and bench experiments agree well with each other.

Good parasitics extraction is key to validate the model.
The crossover frequency is slightly below 10 kHz, good phase margin

This plot confirms the model is representative of the hardware
Transient Tests in Closed-Loop Conditions

- The transient response is excellent and shows a low dc deviation of 1.65 mV.

![Graphs showing transient response with voltage step from 0.5 A to 1 A in 1 µs](image)

- 8.5-kHz crossover with 60° PM
- 9.5-kHz crossover with 45° PM

- It’s impossible to infer margins from transient tests: always go for Bode plot.
Wrongly selecting the crossover shows oscillations in the response.

The system regulates well in dc but cannot fight the oscillations brought by the LC filter: increase crossover!

An output impedance magnitude plot would show the peaking.
A Buck Converter in Current-Mode Control

- The current mode architecture requires a high-side sense

A current transformer references the inductor current to ground
A current sense transformer is now needed to sense $i_L(t)$.

You can toggle between open- and closed-loop configurations.
Current Sense Transformer Operations

- The current is scaled by the turns ratio and generate a sense voltage

\[ V_{\text{sense, max}} = 1 \text{ V} \]
\[ N = 100 \]
\[ I_{D, \text{max}} = 8 \text{ A} \]

\[ R_b = \frac{V_{\text{sense, max}}}{I_{D, \text{max}}} N = \frac{1}{8} \times 100 = 12.5 \Omega \]

\[ R_{\text{sense}} = \frac{R_b}{N} = 125 \text{ mΩ} \]

- The equivalent resistance value is used for small-signal analysis

This resistor value is passed to the model.
The transformer features a magnetization inductance. Reset is mandatory.

\[
V_{mag} D_{max} T_{sw} = V_{reset} \left(1 - D_{max}\right) T_{sw} \quad \text{max V.s}
\]

\[
V_{mag} D_{max} T_{sw} = \left(V_{sense,\max} + V_f\right) D_{max} T_{sw} \left[V \cdot \mu s\right]
\]

\[
V_{reset} = \frac{V_{mag} D_{max}}{1 - D_{max}}
\]

The Zener diode provides the reset.
Watch for the Magnetizing Current

The magnetizing current can affect the precision

\[
v_{\text{sense}1}(t) = i_L(t) R_{\text{sense}}
\]

\[
v_{\text{sense}2}(t) \approx \left[ \frac{i_L(t)}{N} - i_m(t) \right] R_b
\]

\[
\Delta I(\%) = \frac{V_{CS} + V_f}{L_{\text{mag}} DT_{\text{sw}}} \times 100
\]

Check the magnetizing inductance value to minimize this error

\[
V_{CS} \text{ is the maximum voltage sense limit}
\]
The Magnetizing Current Affects Compensation

- The magnetizing current subtracts from the monitored current
- A wrongly-selected current transformer can bring instabilities

On-time slope

\[ S_n = \frac{V_{in} - V_{out} R_b}{L} \]

Compensation slope

\[ S_a = \left[ \frac{V}{s} \right] \]

Magnetizing slope

\[ S_m = \frac{V_f + V_{CS} R_b}{L_m} \]

\[ S_{tot} = S_n - S_m + S_a \]

Can affect slope compensation
Simulation and bench experiments show some discrepancies

- $V_{out}(f)$
- $V_c(f)$

$\angle H = -70^\circ$

$|H| = -24 \text{ dB}$

- Probably a better parasitic extraction is needed from the components
The crossover frequency is slightly above 10 kHz, good phase margin

A large dc gain induces a low output static error
The transient tests confirm a stable converter.

\[ V_{in} = 9 \text{ V}, f_c = 10.5 \text{ kHz}, \text{PM} = 60^\circ \]

\[ C_{out} = 220 \mu\text{F} \]

\[ r_C = 50 \text{ m}\Omega \]

\[ \Delta V_{out} \approx \Delta I_{out} \frac{1}{2\pi f_c C_{out}} = 34 \text{ mV} \]
Instability in Discontinuous Conduction Mode

As $M$ approaches 0.666, the pole goes to lower frequency

Current mode:
$V_{in} = 14$ V
$V_{out} = 5.8$ V
$R_{load} = 80$ Ω
DCM operation

Inject slope compensation to stabilize the converter

Current mode:
$V_{in} = 9.55$ V
$V_{out} = 5.6$ V
$R_{load} = 80$ Ω
DCM operation

$S_a > 0.086 \cdot S_{off}$

The buck converter can be operated with different schemes

- Current-mode control is the most popular technique
- Modeling still sees new emerging techniques
- The PWM switch model is truly the simplest approach
- You can improve models by accounting for harmonics
- The FACTs are an efficient tool to determine transfer functions
- Analytical analysis gets you the insight on who does what?
- Slope compensation is necessary in current-mode CCM
  - but also in DCM for the buck converter to keep stability
- As usual, analytical analysis + simulation + bench prototype = success!